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PERFORMANCE EVALUATION OF THOROUGHLY ADAPTIVE PARTICLE FILTER (TAPF) FOR 3D RADAR TRACKING APPLICATIONS

KADİR GÖKBERK YAPICI

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PERFORMANCE EVALUATION OF
THOROUGHLY ADAPTIVE PARTICLE FILTER (TAPF)
FOR 3D RADAR TRACKING APPLICATIONS

3D RADAR TAKİP UYGULAMALARINDA
TÜMÜYLE UYARLI PARÇACIK FİLTRESİ'NİN (TAPF)
PERFORMANS ANALİZİ

KADİR GÖKBERK YAPICI

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Chairman : Prof. Dr. Hamit ERDEM

Member (Supervisor) : Asst. Prof. Dr. Selda GÜNEY

Member : Assoc. Prof. Dr. Mustafa DOĞAN

APPROVAL

..../09/2019

Prof. Dr. Faruk ELALDI

Director of Institute of Science and Technology

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Tez Başlığı : 3D RADAR TAKİP UYGULAMALARINDA TÜMÜYLE UYARLI PARÇACIK FİLTRESİ'NİN (TAPF) PERFORMANS ANALİZİ

Yukarıda başlığı belirtilen Yüksek Lisans tez çalışmamın; Giriş, Ana Bölümler ve Sonuç Bölümünden oluşan, toplam 89 sayfalık kısmına ilişkin, 27/09/2019 tarihinde şahsım/tez danışmanım tarafından Turnitin adlı intihal tespit programından aşağıda belirtilen filtrelemeler uygulanarak alınmış olan orijinallik raporuna göre, tezimin benzerlik oranı % 6'dır.

Uygulanan filtrelemeler:

1. Kaynakça hariç
2. Alıntılar hariç
3. Beş (5) kelimeden daha az örtüşme içeren metin kısımları hariç

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Onay

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ABSTRACT

PERFORMANCE EVALUATION OF THOROUGHLY ADAPTIVE PARTICLE FILTER (TAPF) FOR 3D RADAR TRACKING APPLICATIONS

Kadir Gökberk YAPICI
Başkent University Institute of Science and Technology
The Department of Electrical and Electronics Engineering

Building 3-D Radar tracking system generally comes with issues of non-linearity on both state and motion model. In this study, several common tracking algorithms are compared performance-wise under noisy environment, mismatched model and unsteady non-linear motions considering application areas such as ground based missile guidance. A radar front end and a space-time adaptive radar data cube is processed in order to achieve realistic observations from target motion which is described as discrete time inputs for tracking algorithms.

After an analogical approach between kalman-based filters, the study focuses on particle filter, which is chosen from mentioned algorithms to be enhanced based on track performance and wealth of the field of study. A thoroughly adaptive particle filter (TAPF) is proposed in order to acquire optimal filtering when the trade-off between degeneracy and impoverishment problems and inverse proportion between over-fitting and divergence, under highly non-linear and noisy environments, are considered. An important sampling proposal with kalman resemblance, which is able to keep track of multiple prior data as a quantization factor, is derived by extending the Bayes theorem on state estimations with processing dependant joint Gaussian noise. Considering the need of regressive information, an effective re-sampling scheme is designed that works in a harmony with both sampling and adaptive particle distribution process based on data likelihood. The ultimate aim of the proposed method is to be able to handle and refine the “intractable”.

Keywords: 3-D Radar Tracking Algorithms, Unscented Kalman Filter, Adaptive Particle Filter, Kalman Resemblance, Maximum a Posteriori Estimation

Supervisor: Asst. Prof. Dr. Selda GÜNEY, Başkent University

ÖZ

3D RADAR TAKİP UYGULAMALARINDA TÜMÜYLE UYARLI PARÇACIK FİLTRESİ'NİN (TAPF) PERFORMANS ANALİZİ

Kadir Gökberk YAPICI

Başkent Üniversitesi Fen Bilimleri Enstitüsü

Elektrik & Elektronik Mühendisliği Anabilim Dalı

3-D Radar takip sistemi kurmak, beraberinde sistem durum ve hedef hareket modellerinde doğrusal olmayan sorunlar yaratır. Bu çalışmada güdümlü füze sistemleri gibi, gürültülü ortamlarda, eşleniksiz model altında doğrusal olmayan haraketli hedefler üzerinde, çeşitli takip algoritmaları kullanılarak performans analizi yapılmıştır. Takip birimlerine gerçek zamanlı hedef radar gözlem girdileri atamak için gerçekçi radar ön uç tasarlanmış ve uzay-zaman adaptif radar veri kübü işlenmiştir.

Kalman bazlı filtreler ile yapılan karşılaştırmanın ardından, çalışma alanındaki zenginliğe ve takip performansına bağlı olarak parçacık filtresi üzerinde çalışılmaya karar kılınmıştır. Buna bağlı, tümüyle uyarlı parçacık filtresi (TAPF) önerilmiş, doğrusal olmayan dönüşümlü ve gürültülü ortamlarda, dejenerasyon, fakirleşme, sapma ve aşırı uyum sorunlarının çözümü hedeflenmiştir. Durum tahminleri için Bayes teoremi, bağıl Gauss gürültüler işlenerek türetilmiş, buna bağlı olarak kalman benzerliğine sahip önem örnekleme önergesi geliştirilmiştir. Bu önem önergesi bir nicemleme faktörü ile geçmiş verilerin getirilerini güncel tutar. Geriye dönük bilgiye duyulan ihtiyaçtan dolayı, örnekleme ve uyumlu parçacık dağıtım işlemi ile uyum içinde çalışan bir yeniden örnekleme planı tasarlanmıştır. Önerilen metodun nihai amacı, işlenmesi ve idare edilmesi zor takip fonksiyonunu, kavrayıp düzenleyebilmektir.

Anahtar Kelimeler: 3-D Radar Tracking Algorithms, Unscented Kalman Filter, Adaptive Particle Filter, Kalman Resemblance, Maximum a Posteriori Estimation

Danışman: Dr. Öğr. Üyesi Selda GÜNEY, Başkent Üniversitesi, Elektrik-Elektronik Mühendisliği Bölümü

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LIST OF SYMBOLS AND ABBREVIATIONS

SYMBOLS

P_T	: Radar power transmitted
G_T, G_R	: Antenna transmit and receive gain
λ	: Wavelength
Rcs	: Radar cross section
S_{min}	: Minimum detectable signal
Range	: Maximum detectable range
R_{max}	: Maximum unambiguous range
c	: Speed of light
τ	: Pulse width
R_{res}	: Unambiguous range resolution
B	: Bandwidth
F1,F2	: Frequency intervals
v_r	: Radial velocity
f_d	: Doppler frequency
k-1, k, k+1	: Previous, current, next tracking time step
F(k)	: State transition matrix
H(k)	: Measurement transition/ Jacobian matrix
v(k)	: Process noise
e(k)	: Measurement noise
Q_k	: Process noise co-variance matrix
R_k	: Measurement noise co-variance matrix
K_k	: Kalman gain
$\dot{x}$, $\dot{y}$, $\dot{z}$	: Cartesian velocities
ω	: Turn rate
$f(X_{k-1})$	: Non-linear state function
$h(X_k)$	: Non linear observation mapping
ψ	: Sigma point scaling factor
α, β	: Sigma point spread factors
x_0, P_0	: Initial estimate and co-variance
x_k, y_k	: Estimations, Measurements

n, N	: number/ total number of particles
ii, i	: State and observation variable sizes
w^n_k	: importance proposal weight
$w^n_{likelihood}$	: Maximum likelihood weight
μ_k	: Mean estimation of particles
s	: Re-sampling time step
N_{eff}	: Effective sample size
σ_k^2	: Mixed Gaussian noise variance
Err	: Error margin
Err_T	: Error margin threshold
$x_{\alpha/2}$	: Confidence Interval
$True_k$	: True values of target motion

ABBREVIATIONS

3-D	: Three dimensional
S.t.d.	: Standard Deviation
EKF	: Extended kalman filter
UKF	: Unscented kalman filter
PF	: Particle filter
UT	: Unscented transformation
KF	: Kalman filter
MAP	: Maximum a Posteriori
TAPF	: Thoroughly Adaptive Particle Filter
STAP	: Space time adaptive processing
RF	: Radio Frequency
RMSE	: Root mean square error
SNR	: Signal-to-noise ratio
PRI	: Pulse repetition interval
PRF	: Pulse repetition frequency
DSP	: Digital signal processing
RDC	: Radar data cube
CPI	: Coherent processing interval
CFAR	: Constant false alarm rate

FIR	: Finite impulse response
CTRV	: Constant turn rate and velocity
FPGA	: Field programmable gate array
LSIR	: Local search importance re-sampling

1. INTRODUCTION

Radar is a system which is used for the purpose of detection and parameter estimation of targets with the help of electromagnetic waves that is emitted, reflected from target directly and received as an echo by radar receiver. 3-D radar systems usually form information about range, azimuth, elevation and Doppler velocity of targets [1]. However, localization success depends on prediction of future values from previous ones. So, one needs a compatible tracking process in order to achieve better results based on the historical discrete time data and estimations that provides a deterministic target trajectory. Target tracking radar systems call upon tracking algorithms, which initiate a near-continuous time track, are required in order to update and estimate true position of a target and derive future position with sufficient precision and accuracy [2].

1.1 Problem Statement and Objective

A target tracking radar provides refinement on predicted expectations and future gating of the target which results with adjusted and corrected trajectory based on performance and purpose of the track processor. Missile guidance for military systems is a suitable motivation source for non-linear, noisy, 3-D Radar system approach as it is mandatory to manage a proper high-quality tracking for it. It is not easy to acquire perfect trajectory due to possible disturbances, natural clutter or electronic counter-measures from the target [3]. By utilizing mentioned motivation source, one can assume sensor sensitivity decays with distance or unwanted system delays which leads to the ideology that implies the significance of enhanced tracking with highly noisy poor measurement data which has known error characteristics. Blair, Richards and Long [4] elaborates on these errors such as system constraints, multipath, calibration errors and various recognized interference and characterize their effect with accuracy and precision. This study focuses on handling the precision of a tracker and improving short-term accuracy mean errors since long-term accuracy errors are considered as systematic which is manageable on system levels. Precision scope is defined as standard deviation (S.t.d.).

For a short range single target tracking radar system, tracking issues include non-linear functions' accurate coordinate conversions, highly non-linear manoeuvring motions, model mismatches and poor measurement precision and accuracy with an expected Gaussian error distribution based on upcoming data with impulse response variation. The objective is to estimate future states of a system based on given noisy sensor outputs and model of dynamics with uncertainties. Chung, Chou, Chen and Chuo [5] uses multiple sensor readings in order to increase accuracy and reliability of non linear-functions with non-linear manoeuvre motions. Multiple sensors provide data association for better coordinate conversions. That being said, most target tracking applications lack the opportunity and have to handle noisy non-precise measurement errors on non-linear functions and dynamics. Widely-known non-linear filters and their algorithms are considered for location correction and estimation such as extended kalman filter (EKF), unscented kalman filter (UKF) and particle filter (PF) according to Konatewski, Kaniewski and Matuszewski [6]. This study focuses on comparison of these filters corresponding to their minimization of process and measurement noises, manoeuvre performance, success on handling with non linear function variables and their moments. Based on error distribution and information obtained, the study majors on developing a new method that satisfy the objectives with enhanced accuracy and precision with minimal divergence and over-fitting on measurements.

1.2 Literature Review

Numerous studies struggle with non-linear tracking filters by enhancing their performance, mutilating the methods for algorithm designs partially or completely or fusing different Monte Carlo and Bayesian tracking techniques in order to optimize posterior predictions of a tracking radar system. Most tracking applications make use of EKF even though it has high linearization errors while dealing with non-linear problems. Mittermaier, Siart, Eibert and Bonerz [7] addresses this problem by creating a multi-sensor environment for short range radars that considers Doppler velocity which makes the localization a non-linear problem. Estimation accuracy is covered with precise models and their stochastic process and measurement properties. Another issue is that EKF's consistency depends on initialization. Precision of estimations is provided with the help of maximum likelihood and data fitting. Results contain 3-D movement characteristics.

Another challenge of EKF is adaptation to manoeuvring targets as distant linearization brings up excessive uncertainties that causes reduced performance, even divergence. Liberato, Pizzingrilli and Longhi [8] introduces model switching via interactive multiple model with EKF banks which has advanced model design and depictions for missile guidance. Quijano [9] suggests a different alternative to EKF and compares it with PF considering smoothness under model mismatch and noisy measurements. The results indicate that EKF's performance is limited with the smoothness of the non-linear function as EKF linearizes it around a single point. Although PF lacks designing of a passable noise model, on sharp edges it has better performance as it estimates second moments of observation errors instead of only first moments. Rigatos [10] approaches the comparison between PF and EKF from noise distribution. PF does not make any Gaussian assumptions on this distribution while dealing with state estimation. It is shown that PF has better performance and wider application choices when sensor fusion is available for measurement gathering. However, it is stated that the developments are in return for computational costs.

One gripping proposal, is to use fast genetic algorithm in order to solve all error problems of EKF with intelligence, is suggested by Hasan and Grachev [11]. Kalman estimations depends highly on state and measurement model co-variance matrices. The study presents a genetic algorithm method to optimize and reduce the variance of tracking error models on manoeuvre of the target in order to acquire real time-tracking.

As EKF has various problems that needs to be solved considering model designations, Obolensky [12] suggests to combine two kalman filtering techniques, EKF and UKF, proposed by Julier and Uhlman, in order to describe Gaussian random error with chosen set of sigma points. The combined filter works with an adaptive varying model that deals with non-linearity of the dynamics while UKF is improving the estimated error to its expectancy. It is represented that UKF has similar working principles with EKF and yields enhanced results under the same adaptations and improved conditions. Roth, Hendeby and Gustafsson [13] test this noise sensitivity on non-linear functions by implementing coordinated turn models for tracking manoeuvring as adaptations to non-linear filters EKF and UKF. Results show that, performance with respect to the mentioned noise sensitivity and parameters, is better in case of Cartesian velocity usage in coordinated turn model for UKF rather that polar velocity. Schubert, Richter and Wanielik [14] take it to another level by implementing more curvilinear models to the UKF system and performing a tracking task that compares the performance of models. This interactive system increases the robustness of the expectations which results with better estimations. This advanced motion models are suggested for applications areas such as two dimensional vehicle tracking.

In 3-D tracking it is more challenging to cover every aspects of motion dynamics with low dimensional models. So, 3-D non-linear tracking filters possess model mismatches. UKF has the ability of precise model-free error estimation. Zhou, Huang, Zhao, Zhao and Yin [15] proposes an adaptive UKF that prevents divergence and over-fitting caused by faulty sensor measurements and model mismatches, resulting in estimation precision. The proposed method originates and adjusts the co-variance matrices of process and measurements noise errors in real time in an adaptive manner. Ge, Zhang, Jiang, Li and Butt [16] designs a similar adaptability by working on time varying uncertain noise co-variances on UKF for target tracking. The method involves deduction of real time measurement noise from the redundant previous measurement residuals based on process noise. It is shown that noise adaptation improves the tracking stability compared to standard naive UKF. Wan and Merwe [17 acquaints machine learning algorithms for dual estimation. It can be depicted as expectation maximization for the Gaussian random variable from system co-variance dynamics for process and measurement errors.

UKF is an optimized filter for non-linear function that almost approaches the performance of an optimal linear system Kalman Filter (KF). Though, it is mostly completed, in other words process and measurement noise optimization is the only working field for improvement. Jwo, Chen and Tseng [18] fuses interactive multiple model estimation with adaptive UKF when there is reliable measurements due to sensor fusion. The results show that the improvement by using interactive multiple model is minimal and the only problem that effects the performance of UKF has been achieved and comes to a saturation point. PF has wider working fields and application areas if certain computational constraints are met with. Chatzi and Smyth [19] suggests and evaluates PF as a comparison for UKF based on efficiency for highly non-linear problems. The method concludes with results that Gaussian mixture PF has more robustness and accuracy compared to UKF for heterogeneous displacement and acceleration sensors.

PF has computational constraints as multiple hypothesis are evaluated at the same time. Lately, these constraints are overcame and PF is getting explored in many application areas. Shu and Zheng [20] presents a performance based comparison between PF and Kalman based filters. The study accepts that PF has superior performance for non-linear and non-Gaussian Bayesian tracking under the assumption of low signal to noise ratio and data rate and its outcome, poor measurement inputs. Mean square error results indicate that the trade-off between performance and computational cost can be minimized by improving the filtering method without any significant computational load. These improvements are implemented by working on known PF problems. Wang, Li, Sun and Corchado [21] mentions about these problems and indicates remaining challenges for PF. Mentioned topics include degeneracy, impoverishment, importance proposal design, computational efficiency and intractable uncertainty caused by poor data defined as measurement to tack challenges. The study implies that uncertain tracking scenarios and complications of analyzing track estimations for future ones, leaves non-solved challenges behind.

PF has many working areas that can be challenged. One of them is to solve degeneracy and impoverishment by controlling the re-sampling procedure. Ignatious, Mageswari and Lincon [22] proposes a variance reduction technique that control particle distribution by interfering particle weights and modifying via a fading factor. This factor can be adapted to re-sampling intervals of the system and manages particle distribution variance. Another way to control information loss is to study on importance proposal. Abbeel [23] lectures on importance sampling and re-sampling methods such as optimal expectations of sequential proposal. The lecture also suggests adapting particle numbers for sampling of particles in order to prevent particle deprivation. Halimeh, Huemmer, Brendel and Kellermann [24] take one step further and combine sequential importance sampling and re-sampling techniques for an evolutionary set of particles selected. The study provides long-term memory on re-sampling stage instead of sampling in order to reduce the effects of degeneracy and impoverishment with computing efficiency. The experiments represents the accuracy and robustness of proposed method compared to standard PF.

Unlike kalman-based filters, prediction and correction stages are applied to multiple hypothesis which compose a grip on complete posterior distribution for estimations. Importance sampling proposals and weighting methods are suggested in order to maximize the performance. Naive PF uses maximum likelihood method as generic for state estimations. Martino, Elvira and Camps-Valls [25] presents group importance sampling with sequential importance re-sampling that jointly employs parallel PF systems. By grouping different schemes, various re-sampling intervals and trajectories are created with independent acceptance probabilities. Though, system complexity increases which is a constraint for PF algorithms. Fu, Wang, Liu, Liang, Zhang and Rehman [26] uses sensor fusion for target localization and calls upon PF and uses sum of Gaussian mixtures of two independent measurements and prior estimation as importance sampling proposal in order to determine posteriori density function. Combined weights of radar and laser sensor measurements decreases the uncertainty based on variance of the particle distribution significantly. Wei, Gao, Zhong, Gu and Hu [27] proposes a different method, unscented particle filtering that adjusts the model noise from predicted residual values. The systems fights with particle degeneracy without losing information on previous estimations by tuning an adaptive factor that uses unscented transformation (UT) to keep system and measurement disturbances minimal. Results claim that usage of UT on PF presents an enhanced performance for navigation systems.

As PF is a rich and practical filter, various study fields are yielded. Data assimilation and kalman techniques have specific weaknesses. On the other hand, PF has a reach on intractable model assignments. Leeuwen [28] benefits from freedom and convenience of importance sampling proposal density to overcome curse of dimensionality, which decreases the efficiency of particles exponentially. The study manages to satisfy high dimensional Lorenz models with low amount of hypothesis for geosciences. In case of tracking variables and their higher moments, state clustering is suggested by Lee and Majda [29]. Instead of standard and localized PF with independent state variables, study benefits from clustering of state variables for particle adjustment that stabilize the distribution of particles. The method presents no divergence and robust results under poor observation gathering regimes. Li, Sun, Sattar and Corchado [30] resorts to artificial intelligence algorithms in order to drawback main problems; degeneracy and impoverishment. effective re-sampling intervals and optimization of particle distribution is suggested with intelligence approach such as swarm or ant colony optimization or genetic algorithm for man-shifting. Filtering in real-life is the main problem of PF combined with intelligent emphasis as more computational cost that multiples for each hypothesis occurs.

Inspiration of this study comes from problems that is encountered, instead of solving techniques. He, Zhang, Hu, Sun [31] touches on one of these problems while working on an adaptive UKF algorithm with adjusted estimations based on maximum a posteriori (MAP) solution. The emphasized problem is determining the balance recursively between co-variance matrices for state and observation models. Usage of maximum likelihood for achieving MAP provides more stable convergence of estimations. Wang, Wang, Li, Wang and Liu [32] presents an adaptive PF method for target tracking estimations. The study focuses on solving deterministic sampling and process noise variance problems with the help of a regression analysis. An auto-regressive model has been designed based on histograms that identify target motion which makes the deterministic iterations stochastic. It is shown that tracking efficiency and robustness is increased via the adaptive model changes. Thus, a new method is derived in this study in order to overcome the challenges with different rustic techniques.

1.3 Methodology

The study includes determination of noisy measurements with varying reliability. Digital signal processing part is featured in order to acquire realistic measurement inputs such as range accuracy and precision by matched filter response for tracking based on radar specifications. Due to the non-linear relation between desired Cartesian output model and spherical observation input model, various non-linear target track estimators are evaluated. These estimators consists of naive formations of extended kalman filter, unscented kalman filter and particle filter. Particle filter is deemed worthy to be worked on depending on its recent prosperous spot in target tracking family and its susceptibility for further performance improvements due to various fields of study on filter's working principle. Particle filter is fixed upon as the focus of the study through kalman based filters for further adaptations.

This thesis contributes with an all-rounded stochastic Gaussian based adaptive particle filter after the consideration of objectives wished to be extended and former literature and studies. The mentioned adaptive methods are linked in harmony via Bayes filtering modifications. Instead of non-Gauss model free PF modifications, kalman resemblance is administered in order to be able to analyze importance proposal outcome and fuse it with re-sampling algorithms. Since there is no co-variance matrix implementations in PF, process noise corresponds to uncertainty added through re-sampling as particle diversion rate. According to these, a combined adaptive importance sampling, state process noise and re-sampling filter is proposed that aims to overcome degeneracy, impoverishment, divergence and over-fitting problems under non-linear/Gaussian noise dynamics based on a well analyzed and handled importance sampling proposal with respect to a standard naive particle filter.

The proposed method is defined as Thoroughly Adaptive Particle Filter (TAPF) since it is designed in a stochastic manner. PF could be designed as model free, but TAPF needs sufficiently accurate state model description in order to acquire reliable expectations based on a MAP similar method and system stability and robustness. As literature review points out, nowadays model constraints could easily be solved for target tracking by covering motion dynamics and their moments.

1.4 Outline

The outline summary of this thesis study is as follows:

Section 2 is the radio frequency (RF) front end design part in which radar fundamentals and working principles are mentioned. Radar parameter specifications are discussed which has effects on significant expressions, that will be taken into considerations for further sections, such as range and Doppler resolution, range and function ambiguity.

Section 3 is the digital signal processing and computing part where Space Time Adaptive Processing (STAP) methods, that is indicated in Figure 1.1, are discussed. The formation and usage of radar data cube is explained with methods such as pulse compression and Doppler processing. Two target scenarios with different motion models are generated in this section corresponding to previous RF front end specifications, signal processing and possible target tracking models.

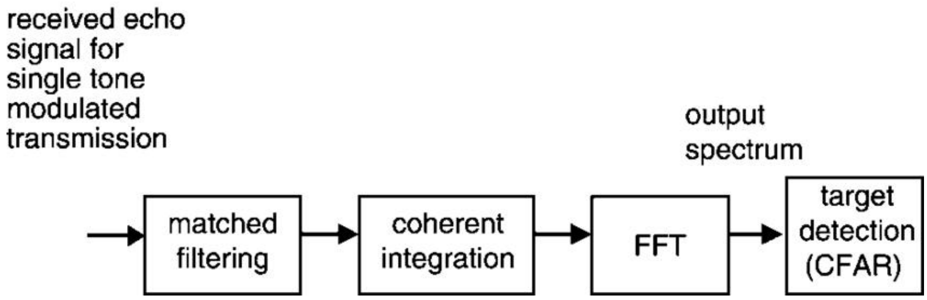

Figure 1.1 Signal processing block diagram

Section 4 consists of analogic evaluation of strengths and weaknesses of tracking techniques which includes the proposed TAPF method. Then, the techniques are compared according to their performance with root mean square error (RMSE) and visual evaluation on critical point estimations. The success of convergence to true mean values without divergence or over-fitting based on the non-linearity of the function or the dynamics when the measurements are noisy and not viable, is represented via the proposed method.

The thesis concludes with foreseeable success of TAPF upon objectives based on comparison between tracking algorithms in Section 5. In case of adapting it to a real time and life application and the challenges of doing it, further improvements are suggested based on attainments acquired during the study.

2. RADAR MODEL

2.1 Radar Fundamentals

Radio detection and ranging, as the term implies, calculate the range of a target from the delayed time between a transmitted pulse and its backscattered energy from the target based on the propagation medium. Designation of RF front end is in charge with waveform generation, amplification, transmission and receiving and filtering of a signal. Signal propagation concept is simply represented in Figure 2.1.

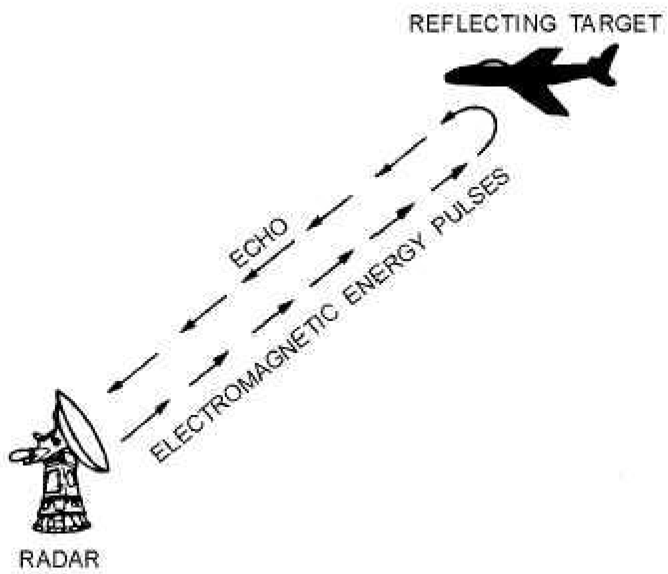

Figure 2.1 Radar concept

Skolnik [33] explains that General Radar formula represents the free space path losses and other target, antenna and radar specifications that clarifies the maximum range which a target can be detectable based on an acceptable signal-to-noise ratio (SNR) over minimum detectable signal. Waveform generation, antenna design and radar parameters are selected according to the desired purpose and performance.

Radar Range equation is as below;

$$Range \cong \left(\frac{P_T G_T G_R \lambda^2 Rcs}{(4\pi)^3 S_{\min}} \right)^{1/4} \tag{2.1}$$

2.2 Front End Design Parameters

The designation purpose of the radar detection system focused on this study, is to work as a short range radar that is capable of gathering radial velocity due to Doppler shift, azimuth and elevation information from a single target. Phased Array antenna systems are able to steer its received pattern digitally for that purpose (Figure 1.3). Uniform Linear array antenna with proper gap between array elements, which can cover SNR with focused directivity, is feasible in common radar systems. Number of antenna elements are proportional to directivity and accuracy of bearing information. S-band as operating frequency encloses surveillance radar requirements.

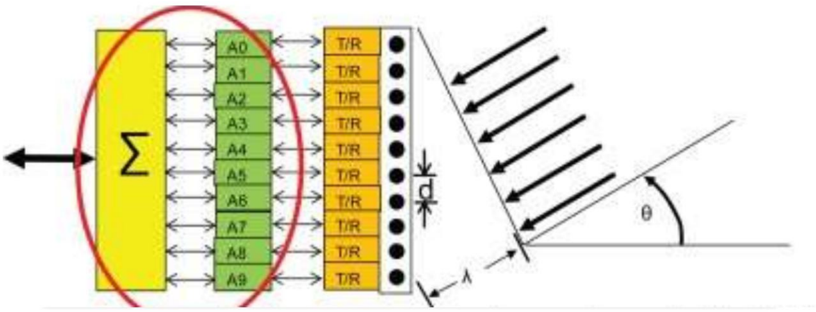

Figure 2.2 Phased array antenna beam directivity [34]

Maximum Range of a radar system is based on both required received power and pulse repetition interval (PRI) which is the inverse of pulse repetition frequency (PRF). Pulse width and PRI of a waveform determine the unambiguous minimum and maximum range respectively. Another issue with waveform design is range resolution as the pulse length increases, the scope it sweeps increases as well resulting with reduced range resolution coverage. The Doppler resolution, which will be mentioned later, is also dependant on PRF value. There is trade-off between all the terms distinguished and should be designed carefully according to the purpose of the system, short range tracking.

Figure 2.3 indicates ambiguous range by representing it in time domain. Unambiguous maximum range equation, where *c* stands for speed of light and *τ* stands for pulse width is as follows;

$$R_{\max} = \frac{c \times (PRI - \tau)}{2} \tag{2.2}$$

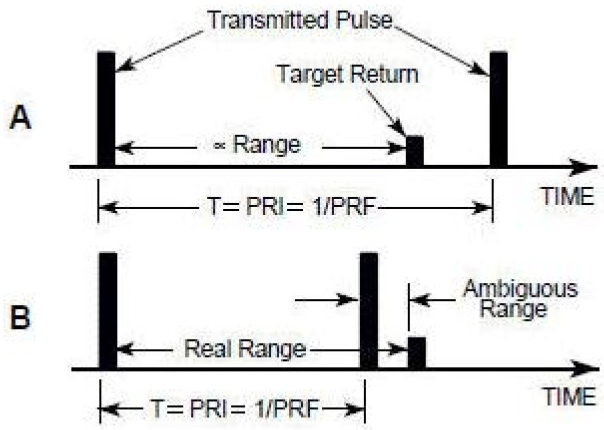

Figure 2.3 Pulse repetition frequency [33]

One wants to detect far objects with better resolution, in other words shortened pulses with more energy. Linear frequency modulated or so called, Chirp waveform satisfies this requirement as modulation of frequency, increases time bandwidth product of the transmitted pulse. This process is called pulse compression (Figure 2.4) and will be mentioned how it is implemented via the matched filter digitally further in the study.

Equation (2.3) represents range resolution for given pulse width while Figure 2.4 explains the bandwidth and pulse width product, where *B* equals to bandwidth that covers the modulated frequency interval between frequency values *F1* and *F2*.

$$R_{res} = \frac{c\,\tau}{2} \tag{2.3}$$

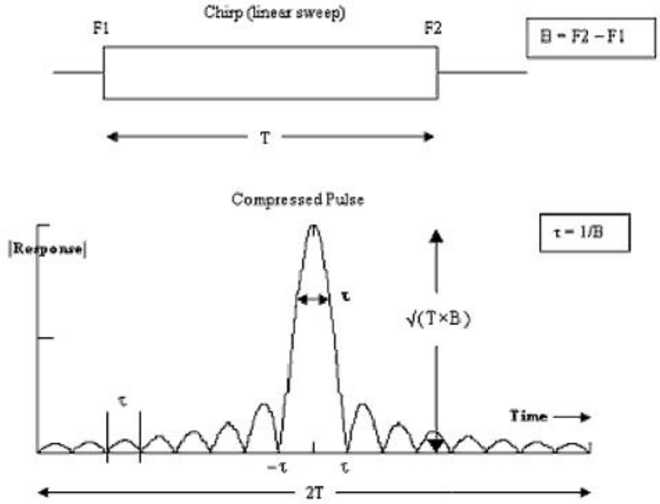

Figure 2.4 Chirp Compression

3. RADAR DIGITAL SIGNAL PROCESSING

Digital signal processing (DSP) is the process where signal are plugged away at and filtered corresponding to various operations that gather information from the message signal. In radar applications Analog to Digital Converters are used to modulate the signal in a way to be ready for digitally processed. High sampling rate is needed in order to acquire near perfect construction of the signal while converting to digital discrete time signal. [34]

The modulated signal is then beamformed, matched to transmitted signal and compressed, shifted in frequency with Fourier transform. Space Time Adaptive Processing can handle these operation and is able to detect targets that is otherwise hard to detect due to background clutter and complications of operations mentioned.

STAP bonds spatial and temporary data and acquires information for real-time processing without any significant latency with the help of a high-dimensional radar data cube (RDC). RDC consists of sampled signal segments, array antenna element bins, storage of multiple consequent pulses that can be processed coherently and a retroactive temporal dimension. Joint storage of mentioned dimensions provides capability of processing signal processing operations along with each other simultaneously.

STAP is used in many airborne radar systems and 3-D ground surveillance of airborne targets as a necessity. However, it has high computational cost that cause latency on the overall system. These constraints of signal processing should be considered as well.

3.1 Generating Radar Data Cube

As mentioned, composing a radar data cube is necessary in order to acquire real-time processing in space-time continuity. It is a convenient way to create storage of data by implementing the signal information in a multidimensional database for further signal processing. The data cube organizes the extraction and gathering of range, velocity and bearing information. In addition to that, accessibility of multiple signal information in the course of space-time continuum provides the capability of decision making during digital processing.

First dimension of a radar data cube consists of range gates which validates a target at a specific range with the designated range resolution due to the travel time of message signal in nature. The derivation of range gates are based on the sampling rate of received signals. Numerous intervals are gathered sample by sample from the reflection of a single pulse which is dependant to PRF. These sampling intervals are binned to successive range values so as to pinpoint the distance of the target. This dimension that includes range bins are referred as fast time dimension in literature due to much higher sampling frequency rather then PRF of the system.

Another dimension is generated which works as an indicator of azimuth and elevation angles. It consists of the collection of a single target reflection in multiple received elements corresponding to array antenna structure. Each antenna element is tied to a specific channel with successively generates a phase difference in collections. This sampling is then used in order to gather accurate bearing information from the target on further STAP processors.

Third significant dimension of the radar data cube is where multiple sequential pulses with the rate of PRF are collected and processed concurrently. The correlation between the coherent received pulses and range gates indicates whether there is truly a target or not and plays a great role on decision-making and initiation of a track. More to the point, the mentioned collection, which is called coherent processing interval (CPI), facilitates the determination of a phenomenon called Doppler effect or Doppler shift. Processing of this shift rate in frequency during the propagation assists on calculation of speed of the target, in this case radial velocity according to radar. This dimension is called the slow time dimension since it is much slower than sampling of PRF and instead composed of multiple pulses. Figure 3.1 represents the dimensions of RDC.

Before working on information of target returns such as range, bearing and velocity with various processing techniques, a threshold must be determined for identification that implies if there is a target or not. When a data exceeds the threshold, a covariance matrix, that is formed by CPI and array antenna element inputs with the help of neighboring range gates, is analyzed in order to get rid of undesired signals' noise and false alarms. [34]

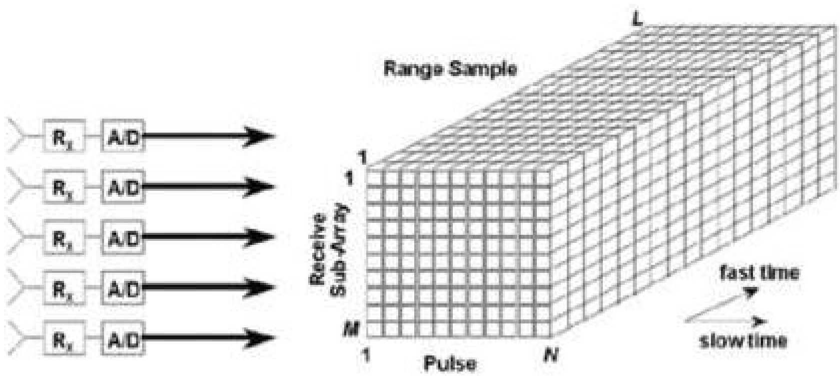

Figure 3.1 Radar Data Cube [34]

In this study, digital beamforming of array elements which includes phase shifts and digital beam steering is considered. Then, pulse compression and Doppler processing techniques are implemented. Direction of arrival estimation is not applied since the focus of the study is track performance and tracking problems of bearing inputs could be assumed realistically. Covariance matrix estimation that shows the correlation between RDC dimensions and Constant false alarm rate (CFAR) algorithms are not implemented in the radar front end and STAP process either, since the focus of the tracking problems does not cover characterization of clutter and reduction of undesired signals by data association and convergence of measurements. This step is assumed as irrelevant since it occurs outside the field of this study. The clutter rejection part is ignored as the study focuses on varying noisy environments. Background clutter and interference is assumed to be settled during the tracking system and algorithms.

3.2 Digital Beam Forming

For an active radar, It is desired to lock up to an area where the possible target is in order to narrow the regarding cut of range gates and reduce the chance of missing the target. In array antenna systems, steering, phase shifting and processing of these are not detached as antenna processing. Considered array's pattern itself can be aimed at the target with phase rotation. Beam steering occurs in azimuth and elevation dimensions. By that way, antenna system ensures only the raw data and beamforming happens digitally by STAP processors.

The digital beamforming module is responsible for determining the directions of the target by creating digital beams. The module runs finite impulse response(FIR) filter with longitude that equals to number of array antenna elements. Each of these FIR elements are pre-allocated to allow formation of a beam on specific special direction. This spatial beamforming allows signals to be amplified only on chosen direction intervals when the signal fall into it. All other directions are suppressed. By that way, mentioned FIR elements are plugged into certain directions without mechanical rotation of hypothetical antenna but with phase shift. The beamforming process is visualized in Figure 3.2.

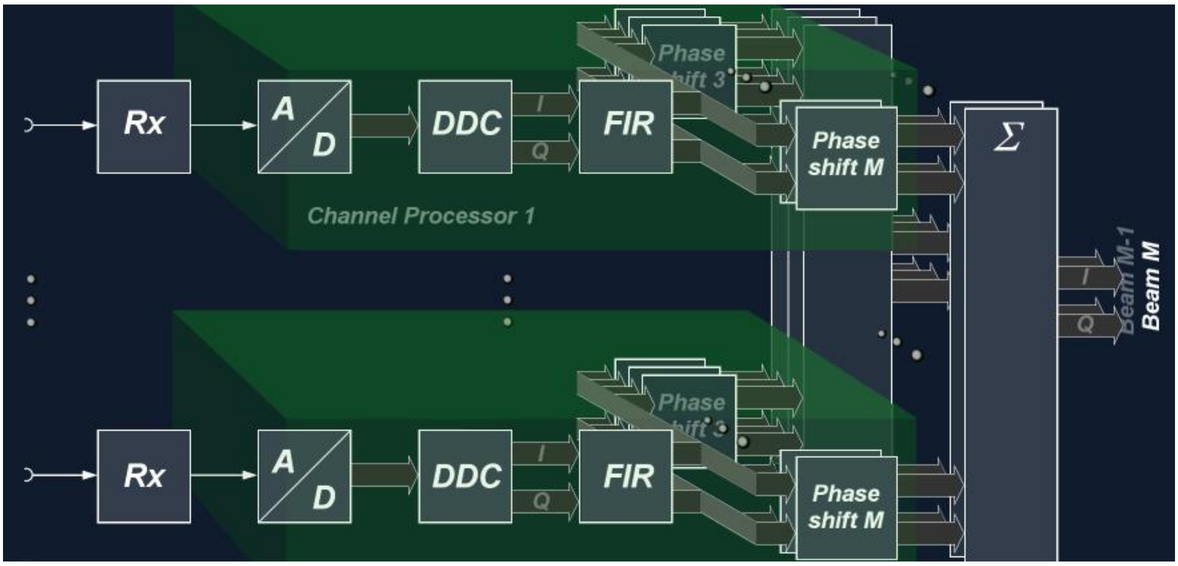

Figure 3.2 Block diagram of digital beamforming [35]

Another advantage of this filtering is that the beamforming process increases the directivity of concerned antenna elements significantly. Increase in directivity effects overall gain and SNR of the system directly by amplifying the received channel outputs.

3.3 Pulse Compression

Pulse compression is a technique that achieves collection of the power during the pulse on a single absolute point as a peak. This process is done by a time domain convolution between received signal from the target and complex conjugate of transmitted message signal. In other words, the process increase the SNR ratio via the matched filter. Transmitted signals from radar system are used as only coefficients for FIR input. By this way, phase of the transmitted signal is ignored and only the target's phase stay online. Peaks are generated on the spots that correlation occurs between these signals.

Simply, the usage of chirp waveform allows the system to use matched filter as a convolution between echo signals and anti-chirp which leads to a compressed near-impulse response as an output in theory. Pulse compression provides better range resolution without trading it off with speed resolution. The idea is to acquire range resolution property of a much shorter pulse by modulating a longer pulse without increasing its function ambiguity for both cases of range and velocity.

An issue of impulse response function is the integrated side-lobes. High side-lobe clutter levels damage the radar sensitivity as it may effect further data. The system should be acquainted with side-lobe suppression in order to obtain better and trusty range resolution. However clutter rejections are out of field and omitted in this study as various assumptions on noise level will be represented. Side-lobe clutters could be easily attenuated with directional selectivity of the array antenna pattern [36].

Range bins are evaluated at this stage so as to determine range of the target based on the time delay. Then, Doppler processing technique is applied on concerned range gates that includes the targets echo. The joint pulse compression and Doppler process is expressed in Figure 3.3.

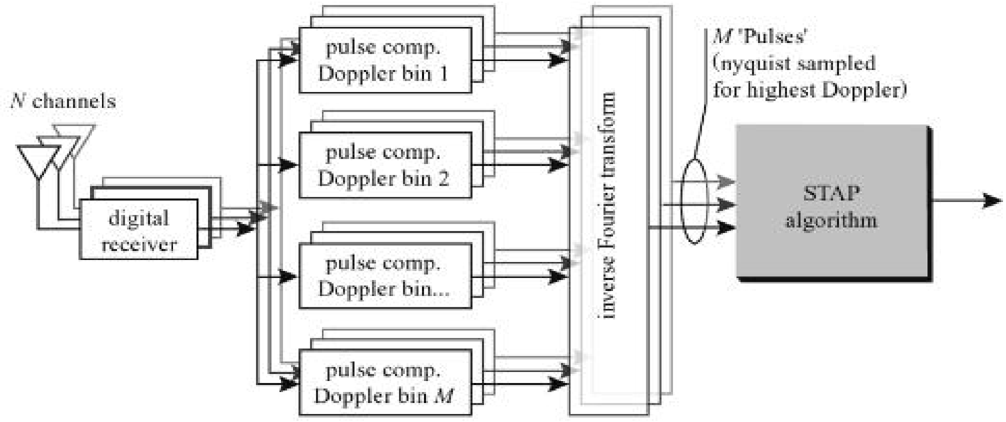

Figure 3.3 Matched filtering of pulse Doppler radar [36]

3.4 Doppler Processing

Working principle of a radar is to perceive and interpret the delay of signals transmitted and received. RDC creates an opportunity of processing multiple fast time data, which has already extracted as gates that the target is within, simultaneously with the help of coherent processing. This coherent processing interval is called slow time and its length via the sampling rate determines the radial velocity resolution.

Fast Fourier Transform is applied to discrete slow time dimension in order to transfer signals from time dimension to frequency dimension. The frequency shift between received echoes of sequential transmitted pulses manifests the velocity relative to the stationary radar which comes up as radial velocity as an inverse function. Figure 3.4 presents Doppler processing along slow time dimension N based on maximum value of fast dimension L.

$$v_r = \frac{f_d \lambda}{2} \qquad (3.1)$$

In equation (3.1), v_r is radial velocity, f_d is doppler frequency and λ is wavelength of the message signal.

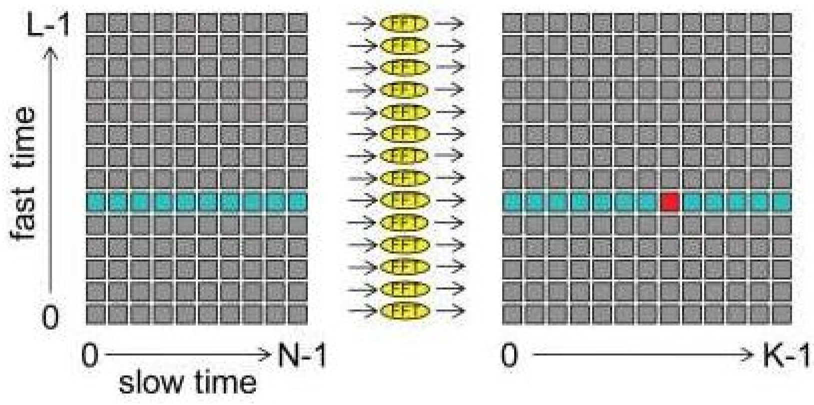

Figure 3.4 Doppler processing along max range bin [34]

As beat frequencies are generated due to Doppler frequency, the velocity resolution and unambiguous velocity range of a target based on velocity bins is directly proportional to PRF that leads to the trade-off between range and speed resolution for certain operating frequency. Increased time intervals based on low PRF between coherent processing elements also limits the ability of detection of a target under clutter since coherent processing also comes in handy for removing stationary or low speed background clutter for an airborne target. So, waveform specifications should be selected carefully based on all design concerns.

Equation (3.2) implies the importance of PRF selection as it determines range of Doppler frequency that can be estimated;

$$PRF = \frac{1}{f_d^{max} - f_d^{min}} \tag{3.2}$$

3.5 Scenario Design

Design environments are used during the study on both RF front end and DSP simulations, and evaluation of tracking algorithms. All simulations, designations and tracking algorithms are produced and tested in these environments starting with design of the scenario. Radar system and signal processing parameters are selected based on desired designated general purpose of the system. The concept is to model tracking algorithms and optimize them for a realistic 3-D short range single target search and track radar system which deals with unwanted noise signals during tracking process.

Following radar and waveform specifications are assessed in Table 3.1 in order to achieve almost real-time simulations, observations and detection errors that feeds the calibration and performance evaluation of studied tracking algorithms;

Table 3.1: Radar design specifications

Operating Frequency	2e9 Hz
PRF	10e3 Hz
τ	1e-5 s
Sampling Rate	10e6 Hz
Number of array elements	100
Element Spacing	0.225 m
Total Antenna Gain	75
CPI	300
Minimum acceptable SNR	15 dB

Constant Turn Rate and Velocity (CTRV) model will be commonly used and discussed during the study in Section 4.2. The model characterizes the yaw movement of possible target onto a simple constant velocity motion model. Model is widely used in two dimensional systems and acceleration moments of the model are considered as the independent process noise. When the model is adapted to three dimensional systems, pitch movement of the target remains as uncertainty.

Two target scenarios has been modeled in order to cover the area of model mismatches and uncertainty degree of tracking state models. First target starts with a 43 seconds of constant velocity motion along a single line with almost irrelevant elevation. Then it makes a severe turn briefly and starts manoeuvring at mild variable rates for 90 seconds. Lastly, it starts accelerating at a constant rate until it falls out of the maximum radar range for 25 seconds (Figure 3.5). Second target makes a helix-wise motion which is jointly centred on both dimensions. It basically tumble mildly laterally (Figure 3.6). The targets become online at maximum unambiguous range 13.5 kilometers.

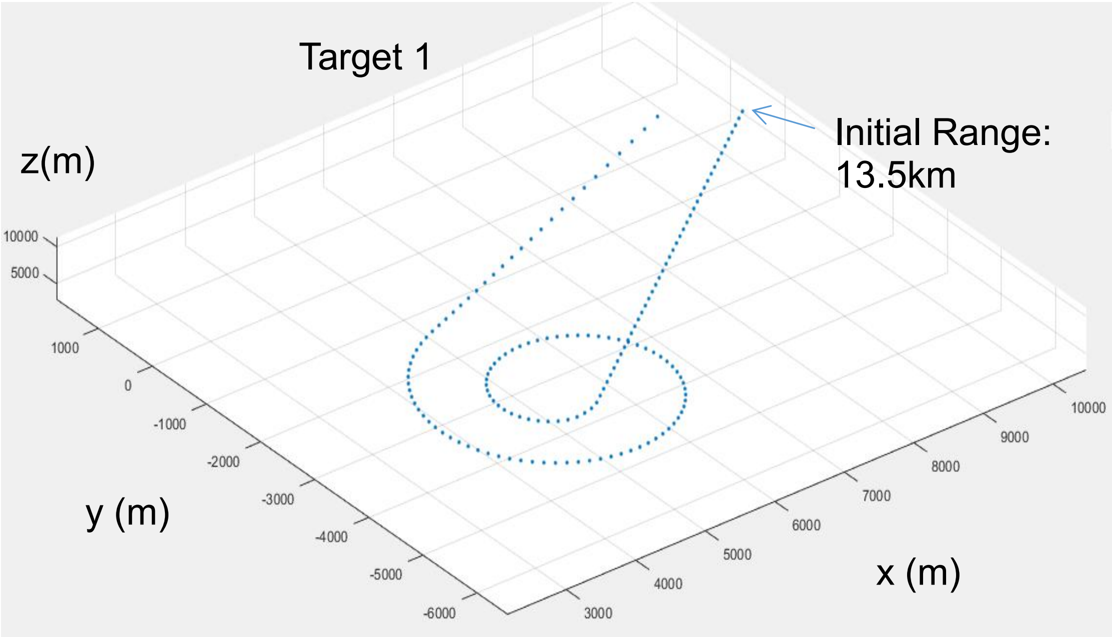

Figure 3.5 Designed motion model of Target 1

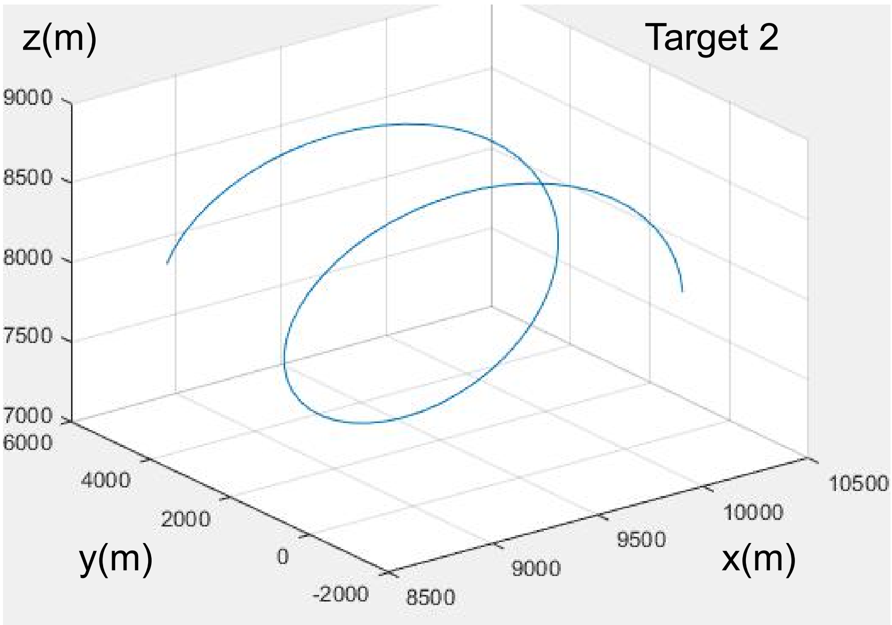

Figure 3.6 Designed motion model of Target 2

DSP affords an opportunity so as to measures for radar outputs, target environment and simulations. The results are yielded in Figure 3.7 for a single pulse.

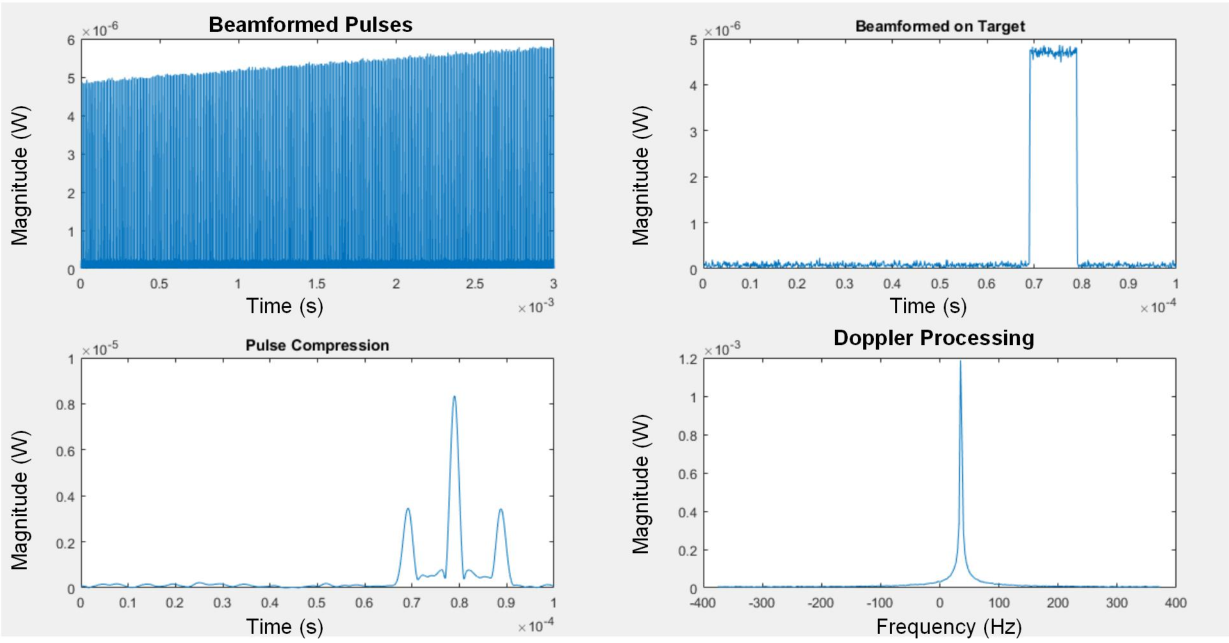

Figure 3.7 Beamforming, matched filtering and Doppler processing outputs for a single pulse

After these steps, interpretation of the outputs of each radar process delivers the desired outputs which includes range resolution that is calculated based on the time gaps between range gates that is 15 meters. A better estimate is derived based on the half power of the output impulse response of compression that gives a theoretical distribution for measurements with a certain variance. On low SNR this value could be deteriorated and become much higher.

Table 3.2 Relevant resulting terms of radar system

Unambiguous Maximum Range	13.5 km
Unambiguous Doppler Range	375 m/s
S.t.d of measurements	50 m
Radial Velocity Resolution	2.5 m/s

Although programming environment is used to create the environment and the simulations of radar system that generates measurements and track initiation, these simulations could be adopted to real time signal processing. In order to satisfy computational load, constraints and requirements of a real-time processing of the mentioned STAP system consists of correlation and white noise generation, Özgür [37] suggests that field programmable gate array (FPGA) could be suggested with its parallelism feature since it consists of only hardware. It can handle programming of multiple arithmetic and computational operators. However, graphic processing unit platforms are preferred due to the ease of processing floating points.

4. TARGET TRACKING ALGORITHMS

A relevant question on tracking radar systems is why one needs a tracking algorithm instead of just initializing and focusing on a detected target. The reason is that tracking radar systems measure the significant parameters which the system then keeps track of by predicting the future values. This predicted state of the relative parameters is corrected based on the recursive process of the concerned tracking algorithm. These attributes are mandatory considering application areas such as airborne localization, active homing, robotics, storm tracking. This study conceives a ground-based flight guidance for calibration of interested tracking algorithms.

The performance of tracking algorithms depends on the validity and precision of generated tracking gate for posterior that the algorithm creates recursively from prior information. An optimal tracker should be able to follow the true motion of an object without diverging from it completely or over-fitting the estimations on input measurements which is given as inexact and noisy observations. RMSE estimations and visual resources on critical stages represents the performance of compared algorithms in this study. Algorithms have a step time of T=1 second for measurement updates.

Methods of tracking involves linear quadratic estimations, linearization of non-linear systems for that matter and sequential Monte Carlo practices. Recursive Bayesian approach is used one way or another in order to gather information on probability of predicted density using existent data. Kalman filter theory, extended and unscented kalman applications of it for the non-linear system, particle filtering and a proposed thoroughly adaptive particle filter which resembles kalman in theory are suggested in this section.

4.1 Kalman Filter

KF is a linear quadratic estimation theorem that can predicts and corrects the posterior estimations of the state at each iteration. What makes kalman filter so special is that it has knowledge on how much predictions and measurements are flawed and incorrect. The linear stochastic system is as follows;

$$x(k+1) = F(k)x(k) + v(k) \qquad v(k) \approx N(0, Q_k) \tag{4.1}$$

$$y(k) = H(k)x(k) + e(k) \qquad e(k) \approx N(0, R_k) \tag{4.2}$$

where x and y are state and observations at time step k respectively. *F(k)* and *H(k)* represents state and observation functions that controls the dynamics of the model. *v(k)* and *e(k)* represents process noise and measurement noise respectively in the dynamic system. Q_k and R_k are their Gaussian covariance matrices that is defined as additive white noises to system.

State space model of a kalman filter consists of a state process, its independent process noise and a joint observation model with an independent measurement noise. Kalman filter predicts and corrects based on a kalman gain which is derived from the gaussian distribution of prior estimation and independently from the state estimation by corresponding co-variance matrices. The mean values and their distributions based on these error estimations anticipate the distribution of a posterior estimate which leads to searching of the best solution at each iterative step.

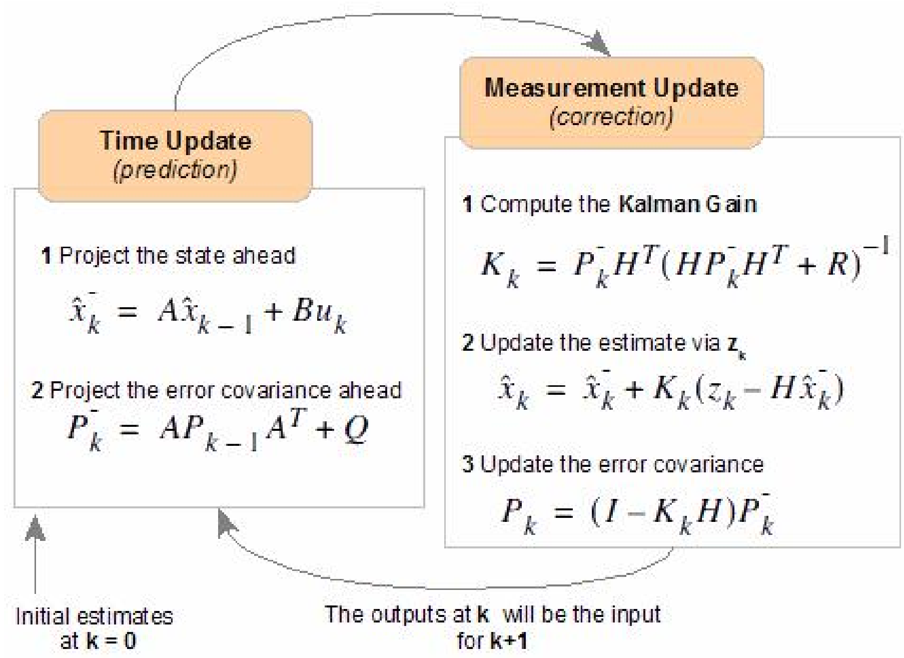

Figure 4.1 Kalman filter sequel [38]

Figure 4.1 is formed of Riccati equation derivations that corresponds to prediction and correction stages of kalman filter. K_k is the kalman gain that tunes and minimizes the error co-variances for future estimations where k indicates the current track step. The representations of variables that is explained in Figure 4.1 is as follows:

$k-1$ previous time step, state estimates x_k and x_{k-1}, state transition function A, Control function B with control input u_k, error co-variance matrix P, observation function H, process and measurement noise co-variances Q and R respectively. In correction phase posterior state estimate is updated based on prior state estimate that is determined during prediction phase, kalman gain and innovation residual (z_k-H*x_k) where z_k is actual measurements

KF is almost flawless and optimal for cases that obtain linear functions and gaussian distributions around it. However most tracking radar systems consist of non-linear functions due to spherical measurements while one needs cartesian mapping instead of curvilinear outputs. So, kalman filter is well out of the field as one needs to make better assumptions on error estimations since the mean and variance of the function outputs are no longer gaussian.

4.2 Extended Kalman Filter

Kalman filter is unable to calculate the mean and variance values of possible distribution of a non linear function, in this case different observation and state coordinate models. Linear approximation is required in order to estimate utilizable gaussian approach which is achieved from the first order derivative of Taylor series applied on estimations. This process is referred as extended kalman filter (EKF) as it offers an extension by linearization to formulation and calculation of kalman state, observation function and corresponding covariance matrices.

Figure 4.2 mentions about linearization errors of EKF when the function grows apart from the mean. p(x) and p(y) is probability density functions, before and after non-linear transformation respectively while g(x) is the approximated transformation function. Right-hand histogram implies on the increased mean divergence error that is caused by poor transformation.

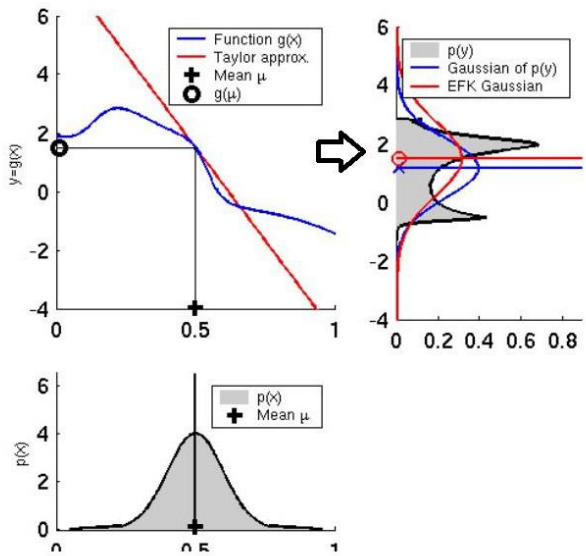

Figure 4.2 Gaussian approximation of EKF linearization [39]

As the measurements consists of azimuth, elevation, range and its first moment, they need to be transformed into “x-y-z” positions and their higher moments for state estimations. Since linear approximation is not accurate, one needs a proper state transition model matrix in order to specify significant motion parameters. CTRV comes in handy as it extends Cartesian position and velocity model with a yaw measurement that is predicted and merged inside the state transition function. CTRV is a consistent model in two dimensional systems. It can be applied in 3-D motion model due to its effectiveness for manoeuvre with fairly low dimensions, with model mismatch that can be handled. It is known that, usage of Cartesian velocity instead of polar velocity in state transition function results with better approximations. [40]

State Vector and Transition Matrix:

$$X = [x, y, z, \dot{x}, \dot{y}, \dot{z}, \omega]' \tag{4.3}$$

For equation (4.3), (4.4), (4.5), (4.6), state variables x, y, z are Cartesian positions. $\dot{x}$, $\dot{y}$, $\dot{z}$ are respective velocities and ω is independent turn rate. *T* is time step interval that is designed as 1.

$$f(X_{k-1}) = \begin{bmatrix} x + \frac{\dot{x}}{\omega}\sin(\omega T) - \frac{\dot{y}}{\omega}(1-\cos(\omega T)) \\ y + \frac{\dot{x}}{\omega}(1-\cos(\omega T)) + \frac{\dot{y}}{\omega}(\sin(\omega T)) \\ z + \dot{z}T \\ \dot{x}\cos(\omega T) - \dot{y}\sin(\omega T) \\ \dot{x}\sin(\omega T) + \dot{y}\cos(\omega T) \\ \dot{z} \\ \omega \end{bmatrix} \tag{4.4}$$

Tracking algorithms in this study are based on CTRV model for precise comparison between their performance under certain circumstances. Augmentation of a turn rate provides tracking of highly non-linear dynamic target model without increasing the state dimensions excessively.

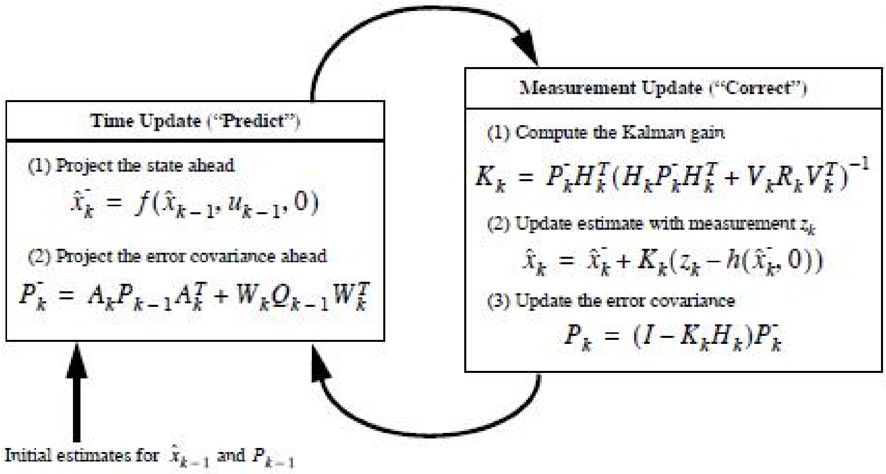

Figure 4.3 EKF Formulation [38]

Figure 4.3 shows that EKF differs from KF based on computation of jacobian matrices H_k based on *f* and *h* function derivatives. that is derived from the coordinate transformation between cartesian and spherical. Jacobian matrix is the computation derived from the Taylor series that deals with the linear approximation. Jacobian computations are responsible for transformation of noise covariance matrices in order to relate them. Prior predictions are mapped to spherical coordinates, which is called the innovation residual part, and posterior estimations are gathered with linearization between state transition and observation functions. As the residual mapping is not one to one, Kalman gain does not control a portion of the system and divergences are expected when the model probability decreases.

Measurement mapping vector and derived Jacobian matrix for CTRV respectively:

$$h(X_k^-) = \begin{pmatrix} \rho \\ \theta \\ \varphi \\ \dot{\rho} \end{pmatrix} = \begin{pmatrix} \sqrt{x^2+y^2+z^2} \\ \arctan(z/(\sqrt{x^2+y^2}) \\ \arctan(y/x) \\ \dfrac{x\dot{x}+y\dot{y}+z\dot{z}}{\sqrt{x^2+y^2+z^2}} \end{pmatrix} \tag{4.5}$$

$$H_k = \begin{matrix} \dfrac{x}{\sqrt{x^2+y^2+z^2}} & \dfrac{y}{\sqrt{x^2+y^2+z^2}} & \dfrac{z}{\sqrt{x^2+y^2+z^2}} & 0 & 0 & 0 & 0 \\ \dfrac{xz}{(\sqrt{x^2+y^2+z^2})^2\sqrt{x^2+y^2}} & \dfrac{yz}{(\sqrt{x^2+y^2+z^2})^2\sqrt{x^2+y^2}} & \dfrac{-\sqrt{x^2+y^2}}{(\sqrt{x^2+y^2+z^2})^2} & 0 & 0 & 0 & 0 \\ \dfrac{-y}{x^2+y^2} & \dfrac{x}{x^2+y^2} & 0 & 0 & 0 & 0 & 0 \\ \dfrac{-((z\dot{z}+y\dot{y})x-\dot{x}z^2-\dot{x}y^2}{(\sqrt{x^2+y^2+z^2})^{3/2}} & \dfrac{-((z\dot{z}+x\dot{x})y-\dot{y}z^2-\dot{y}x^2}{(\sqrt{x^2+y^2+z^2})^{3/2}} & \dfrac{-((x\dot{x}+y\dot{y})z-\dot{z}y^2-\dot{z}x^2}{(\sqrt{x^2+y^2+z^2})^{3/2}} & \dfrac{x}{\sqrt{x^2+y^2+z^2}} & \dfrac{y}{\sqrt{x^2+y^2+z^2}} & \dfrac{z}{\sqrt{x^2+y^2+z^2}} & 0 \end{matrix} \tag{4.6}$$

In equation (4.5), ρ is range, θ is elevation, Φ is azimuth, $\dot{\rho}$ is radial velocity.

Although EKF is widely used for non-linear systems, it is not close to being an optimal estimator for target tracking unlike KF. Approximations are inaccurate due to capturing only the first moment of the terms as linearization happens on a single point. Therefore, the system prones to diverge under bad design parameters, mismatched model or poor quality observations. One of the possible solutions is to use a bank of EKFs with varying state models and uncertainties based on process noise with a likelihood estimation between them using an interactive multiple model that covers all the possible dynamic changes of a motion model. Based on miscalculated means, initial estimations should be close to true values or else they should be adapted by optimization techniques.

4.3 Unscented Kalman Filter

UKF is designed simply to obtain better performance on non-linear functions by approximating almost a true gaussian around the mean of estimations by dealing with a bunch of points instead of transforming around a single point. Since Taylor series expansion terms increase exponentially, third order approximation from its derivatives give nearly perfect essential results. UKF manages that without any linearization process delay for predictions and their covariances.

UKF attempts to structure an optimal KF for non-linear functions. It benefits from sigma points in order to handle approximation of Gaussian plantation with UT instead of sub-optimal first order linear EKF approximation. Sigma points are the towering individuals that represents whole distribution. Certain points are taken into consideration at state coordinate system which manages initial source Gaussian error. Weights are assigned to these points around the mean. Then, these points are propagated mapped through measurement function and a new Gaussian is composed from weighted sigma points. New attributes of the transformed Gaussian are approximated such as mean and variance [41].

Number of sigma points, that scale the dimensions of state estimate, is derived as 2n+1 considering "n" denotes the number of state model dimensions. "X" is the sigma point matrix in this case.

$$\begin{cases} X_0 = \mu_x \\ X_i = \mu_x + (\sqrt{(n+\psi)P})_i \quad for \quad i = 1,...,n \\ X_i = \mu_x - (\sqrt{(n+\psi)P})_{i-n} \quad for \quad i = n+1,...,2n \end{cases} \tag{4.7}$$

$$\psi = n(\alpha^2 - 1) \tag{4.8}$$

w_i represents weights of corresponding sigma points, ψ is scaling factor, μ_x is priori mean and *P* is priori co-variance matrix for equations (4.7), (4.8) and (4.9). These sigma points are propagated through non-linear function separately.

Scaling factors for sigma points, α and β represents the spread intervals and distribution specifications of the sigma points respectively. β=2 is designed as optimal for Gaussian distribution. Sum of weights of the sigma points are equal to 1 and calculated as;

$$\begin{cases} w_0^m = \dfrac{\psi}{n+\psi} \\ w_0 = \dfrac{\psi}{n+\psi} + (1-\alpha^2+\beta) \\ w_i = \dfrac{1}{2(n+\psi} \qquad i = 1,...,2n \end{cases} \tag{4.9}$$

Then, new mean and co-variance should be estimated by multiplication of weights and projected sigma points based on CTRV for corresponding dimensions. Then, the outcome is relocated to measurement space from state. State and measurement functions that is implemented for these calculations are same functions that are mentioned in previous EKF section.

Figure 4.4 indicates the near-perfect approximated error co-variance *P* throughout the non-linear function *g(x)* with better results than linearization and less hypothesis is used called sigma points.

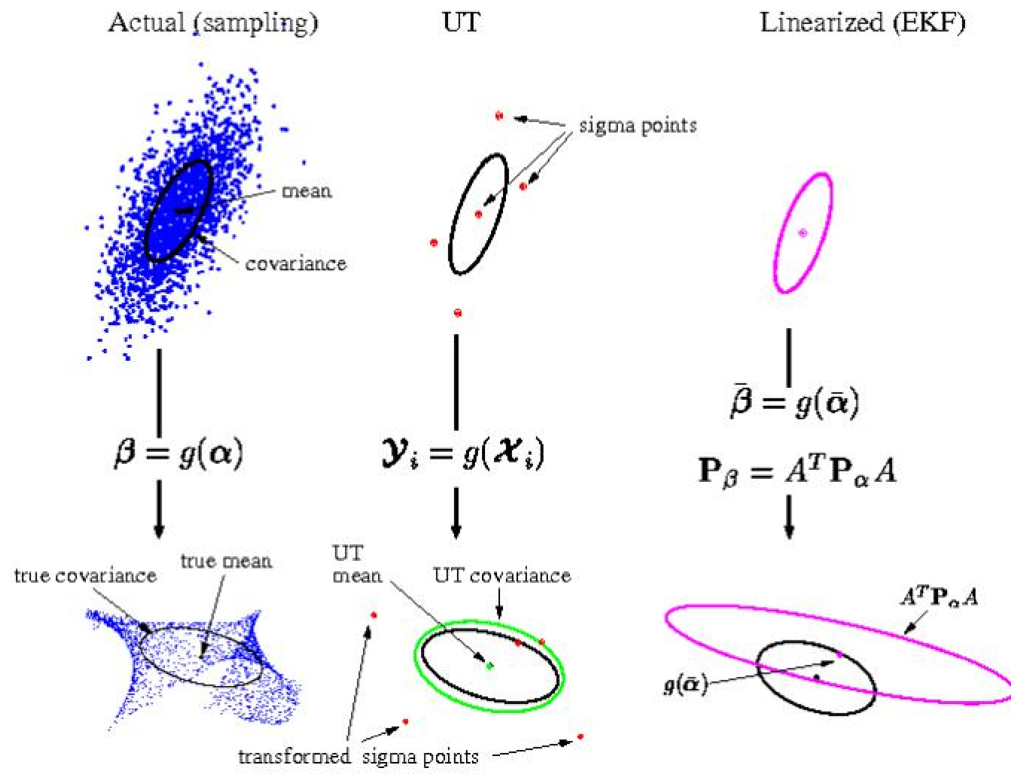

Figure 4.4 Unscented transformation covariance accuracy [42]

Instead of Jacobi computations and linearization, UKF calculates the prediction error via the cross-correlation between the locations of sigma points around the mean in state space and measurement space. The resulting computation kalman gain is similar to the one in EKF. “T” represents the cross-correlation instead of linearization and “Q” represents measurement noise in the following Figure 4.5.

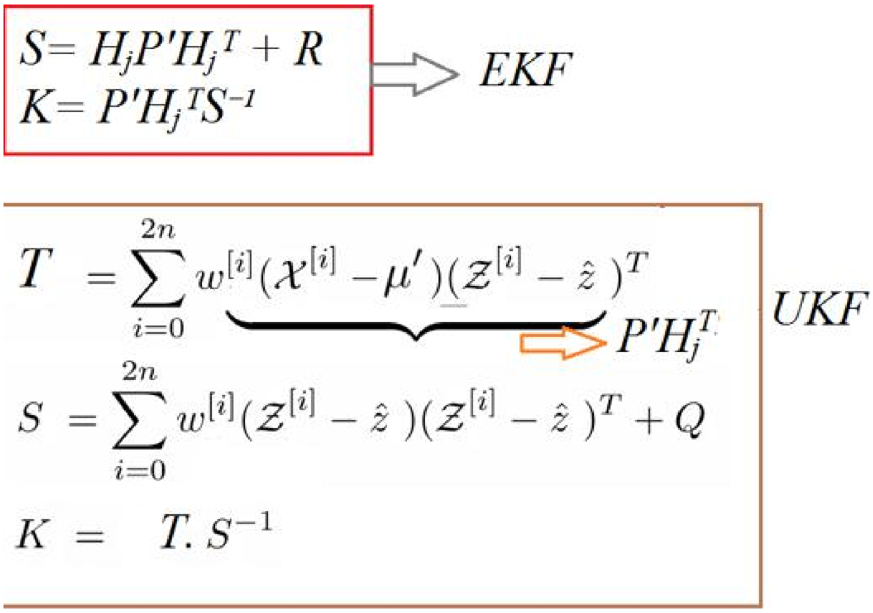

Figure 4.5 UKF correction compared to EKF [38]

UKF has optimal Gaussian approximation for non-linear functions. Computational cost is no more than EKF with better performance. Unreasonable model mismatch can theoretically be handled by uncertainty characteristics. Adaptive process noise estimation based on state circumstances can be implemented in order to get better stability, convergence, smoothness. Matching degree of process noise distribution based on prior information or uncertainty co-variance matrix estimation at present time are examples of study topics that has been implemented in literature as adaptive UKFs.

Although UKF supplements the desired purpose which is tracking, Han Song and He [41] mentions that UKF has a narrow working field for improvement on any aspects. Selected inputs that completes UKF to a closed system are sigma variates; α, β, process and measurement noises; Q_v, R_e, and initial estimates; x_0, P_0. Initial estimations converges with the increasing number of recurrence, UT parameters have negligible effects on estimation accuracy and precision since they are related to higher order terms of derivation and could be calibrated optimally beforehand. Q_v is the only parameter which can be profitable on further performance measures as R_e is stochastic and depends on known measurement error or clutter patterns. If priori or deterministic knowledge of noise exceeds the limits and mismatches with the respective dimensions in state space, the system suffers from degradation caused by lost stability. Otherwise, noise is added the system which causes over-fitting on faulty measurements. This performance issue is the one of the few field that may be improved by mentioned parameters.

4.4 Particle Filter

Particle Filter are based on Monte Carlo sampling method on Bayesian network in order to achieve better approximations on complex dynamic systems. Nowadays computational and physical restrictions are costly but manageable with parallel computing on hardware architectures such as FPGA which brings the particle filter applications back for high dimensional systems.

Multiple particle are systematized instead of a single hypothesis in particle filter. It can deal with non-Gaussian noise output directly, as multiple hypothesis represent the posteriori joint error probability distribution by shaping it. Range of source Gaussian is maximized by the repeated reproduction of particles. In this study, CTRV is suggested to particle filter as a transition model. Model works with 1000 samples which is assessed as enough for a non-clustered dynamic model with 7 dimensions.

This section of the study stresses on naive particle filter theory and a simple implementation of particle filter. PF is known to be distressed by particle degeneracy and impoverishment. Degeneracy is caused by likely particles getting bigger weights and bad assumptions diminishing. The loss of control group is solved with re-sampling which causes impoverishment caused by lost information on particle states. Design details and upcoming problems are discussed at next section.

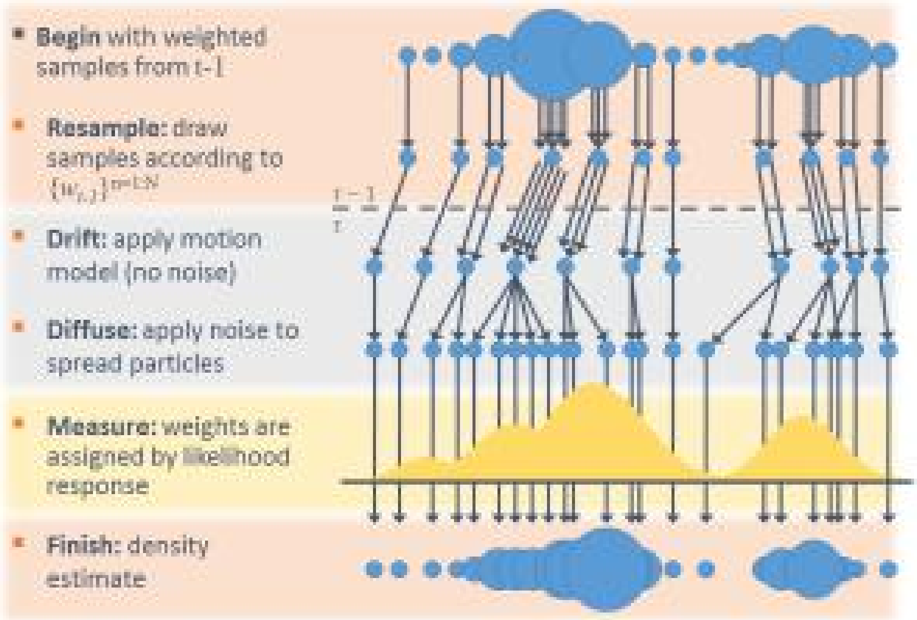

Figure 4.6 Naive particle filter steps [43]

First, state motion model is applied to initial equally weighted and distributed particles. Correction segment of particle filter includes important sampling proposal which evaluates every particle due to their probabilistic occurrence likelihood and weights them. Sub-optimal maximum likelihood and a Gaussian framing are imposed as the sampling proposal that leads to state estimation which is the weighted mean of the distribution outcome of weighted particles. Lastly, a sequential importance re-sampling process is applied on weighted particles in order to obtain new unbiased particle distribution with normalized weights based on process noise. The cycle continues with motion model drift again (Figure 4.6).

$$\left[\{x_k^i\}_{i=1}^N\right] = PF\left[\{x_{k-1}^i\}_{i=1}^N, z_k\right]$$

FOR $i = 1:N$

– Draw $x_k^i \sim p(x_k | x_{k-1}^i)$

– Calculate $\tilde{w}_k^i = p(z_k | x_k^i)$

END FOR

Calculate total weight: $t = \text{SUM}\left[\{\tilde{w}_k^i\}_{i=1}^N\right]$

FOR $i = 1:N$

– Normalize $w_k^i = t^{-1}\tilde{w}_k^i$

END FOR

$$\left[\{x_k^i, -, -\}_{i=1}^N\right] = \text{RESAMPLE}\left[\{x_k^i, w_k^i\}_{i=1}^N\right]$$

END

Figure 4.7 Simple particle filter algorithm [44]

Figure 4.7 shows the iteration of standard particle filter. *N* is number of particles, *k* is time step as initial estimates for each particle x_k^i are propagated through model. Then, a weighting process of particle probabilities occur solely on maximum likelihood of predictions based on measurements, $p(z_k|x_k^i)$. Weights are normalized to a sum of 1 and re-sampled based on specified scheme in order to keep particle amounts alive.

Although particle filter obtains good results based even with a sub-optimal designation, its real-time tracking ability without latency is poor due to the number of hypothesis. However, it is ripe for development as it has the ability to produce probabilistic approximation of whole posterior density function and observe it analytically for further improvements and lots of deterministic design parameters that could be adapted jointly.

4.5 Proposed Method: Thoroughly Adaptive Particle Filter

Sequential Monte Carlo algorithms provides complete information on posterior density function for a Bayesian framework. The context gives opportunity to make statistical inferences from the predicted output. Importance sampling proposal of TAPF algorithm resembles kalman-based filtering and keeps track of prior tractable predictions in order to refine posterior estimations and converge them to true mean using Gaussian framing. Particle filtering has the option to represent full posteriori density function with multiple hypothesis and handle intractable Bayes filter equations. Kalman resemblance creates an opportunity to analyze and benefit from created posterior density characteristics that is applied with Gaussian laws and statistics. Mutual relationship of sampling and re-sampling processes are defined with an error margin factor. This factor that arbitrate the regression ratio of the system, is derived from Gaussian posterior estimation laws and statistics,

One purpose of improved importance sampling method is to prevent divergence when the particles are strict due to determined low additive noise. Other purpose is to forestall overfitting, which is convergence to uncertain measurements due to low trust on suggested model. Adaptive re-sampling, process noise computation, marginalization of correlation between joint Gaussian are implemented as a MAP design with an interconnection between mentioned methods in order to achieve expectation maximization.

The proposed TAPF method contemplates a system environment in which the algorithm manages to handle the quantization step, which is a coefficient factor for importance proposal, based on retrospective re-weighting. The main problem that comes up with the multiple hypothesis computation is that particle degeneracy. High probability particles keep getting heavier with reliable measurements as low probability particles diminish which disturbs the clarity of posterior distributions.

Re-sampling step occurs in particle filters in order to distribute the particles again. However information is lost with the abandoned particles and distribution pattern based on impoverishment of changing particle span and weight at conventional importance re-sampling methods. In order to reduce the dependency of inverse ratio from trade-off between degeneracy and impoverishment, local search importance re-sampling (LSIR) is applied. The output distribution is handled with adaptive process noise that takes the noise of nearby previous measurements as basis.

4.5.1 Importance Sampling Proposal

Importance sampling proposal is a variance reduction method that determines the hypothesis that has more impact than others. KF is successful at handling process and measurement noise dependency that is derived from design of discrete time model. The additive Gaussian noise sequences are assumed as independent themselves. However, there is dependency between steps of them. PF is unable to relate this dependency as a general solution. [45] So, importance sampling proposal function is optimized with Gaussian mixture based on the chosen dependency type that is shown in the following Figure 4.8.

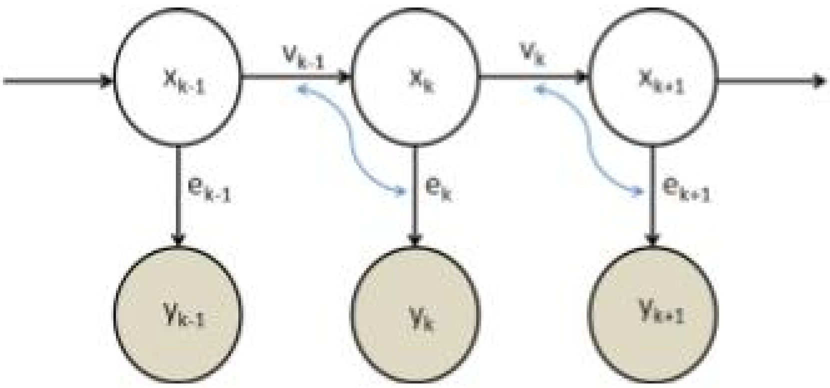

Figure 4.8 Noise dependency of recursive system [45]

State and observation functions consist of CTRV model transformations is given as;

$$x_k = f_k(x_{k-1}) + v_{k-1} \tag{4.10}$$

$$y_k = h_k(x_k) + e_k \tag{4.11}$$

The state space equations x_k (4.10) and y_k (4.11) indicates the state and observation that determines the posterior distribution. During the segment *k-1* and *k* represents previous and current time steps respectively that associates the additive process and measurement noise dependency v_{k-1} and e_k. f_k and h_k consists of non-linear state and observation functions of CTRV model.

Structure of the dependency is as follows where the probability function gives the density shaped with multiple hypothesis variables *X* and *Y*:

$$p(x_k \mid X_{k-1}, Y_{k-1}) = p(x_k \mid x_{k-1}) \tag{4.12}$$

$$p(y_k \mid X_k, Y_{k-1}) = p(y_k \mid x_k, x_{k-1}) \tag{4.13}$$

Equations (4.12) and (4.13) gives conditional probabilities for the mentioned state and observation functions based on the noise dependency. It is shown that measurement likelihood is dependent to both prior and current predictions in equation (4.13).

If we characterize the probabilities of information on previous and present estimations with present measurement with respective additive Gaussian noises, it allows the decomposition (4.14) from (4.12) and (4.13).

$$p(v_{k-1}, e_k) = p(e_k \mid v_{k-1}) p(v_{k-1}) \tag{4.14}$$

Based on this decomposition of the mentioned state and measurement noise dependency, the relation between iteration steps is derived as:

$$p(x_k, y_k | x_{k-1}) = p(x_k | x_{k-1}) p(y_k | x_k, x_{k-1}) \tag{4.15}$$

As Gaussian resemblance is established, corresponding Bayesian theorems (4.16), (4.17), (4.18) are practicable.

$$\frac{p(AB)}{p(B)} = p(A|B) \tag{4.16}$$

$$p(A|B) = \frac{p(B|A)p(A)}{p(B)} \tag{4.17}$$

$$p(B) = \sum p(B|A)p(A) \tag{4.18}$$

x_{k-1} condition that represents the priori data is collaborative for each element for the decomposition equation derived from noise dependency (4.15). The condition is simply met by propagating prior samples through the state model. Using mentioned Bayesian theorems on $p(x_k,y_k|x_{k-1})$, considering A and B incidents correspond to x_k and y_k functions respectively, one can achieve equation (4.19) where n is added in order to represent the samples that shapes the probability density function that they are within and k is time step.

$$p(x_k | x_{k-1}^n, y_k^n) = \frac{p(y_k | x_k^n, x_{k-1}^n) p(x_k | x_{k-1}^n)}{p(y_k | x_{k-1}^n)} \tag{4.19}$$

Where;

$$p(y_k | x_{k-1}^n) = \sum p(y_k | x_k^n, x_{k-1}^n) p(x_k | x_{k-1}^n) \tag{4.20}$$

In equation (4.19), $p(x_k|x^n_{k-1},y^n_k)$ is the proposal density function that is quantized with prior density functions (4.20) which gives the weights to each sample based on their importance determined by the equation. $p(y_k|x^n_k,x^n_{k-1})$ is maximum likelihood of samples based on measurements. $p(x_k|x^n_{k-1})$ is predicted state based on priori estimation. $p(y_k|x^n_{k-1})$ is quantization factor that indicates data likelihood that determines the reliability of measurement y_k at time step k.

Since the observation model is multi-dimensional and kalman resemblance is desired, a multivariate Gaussian model is implemented as for maximum likelihood framing. Weights of samples that shapes maximum likelihood $p(y_k|x^n_k,x^n_{k-1})$ is represented as $w^n_{likelihood}$ in a Gaussian window. R is measurement noise covariance, y^{-n}_k is samples that is propagate and transformed throughout the model in equation (4.21).

$$\widetilde{w}^n_{likelihood} = \frac{1}{(2\pi)^{i/2}}|R|^{-1/2} e^{(-1/2*(y_k-\bar{y}^n_k)R^{-1}(y_k-\bar{y}^n_k)^T)} \tag{4.21}$$

Quantization factor in random process is meant to be a coefficient factor for probability functions. In case of particle filters with multiple hypothesis, quantization shapes the posterior density and determine the balance between state estimations and measurements.

A standard PF with sequential importance re-sampling and sub-optimal sampling has performance flaws and mismatches due to weighting based on maximum likelihood of an uncertain measurement. Since unbiased re-sampling resets the information on prior state estimation weights for each hypothesis, PF tends to lose grip with each particle distribution. Figure 4.9 implies loss of information on prior estimations with each iteration based on particle effectiveness.

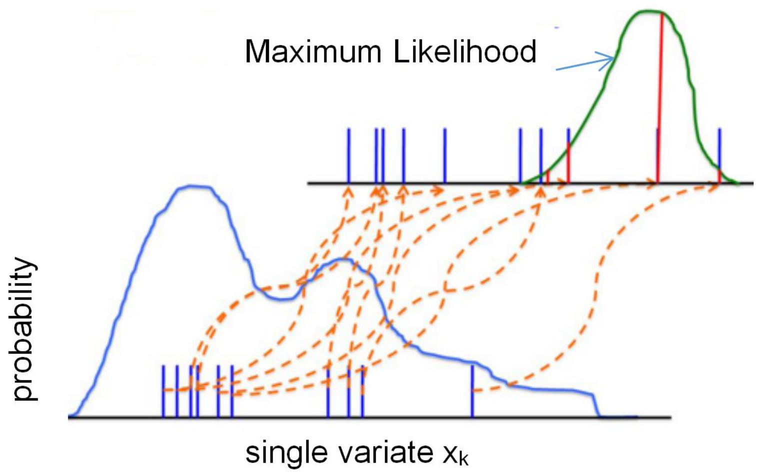

Figure 4.9 Sub-optimal importance sampling

Sub-optimal quantization as a denominator that cancels prior probabilities that leaves maximum likelihood, $p(y_k|x_k^n)$ as solo importance parameter in standard particle filter is given below:

$$q(x_k|x_{k-1}) \propto p(x_k|x_{k-1}^n) \tag{4.22}$$

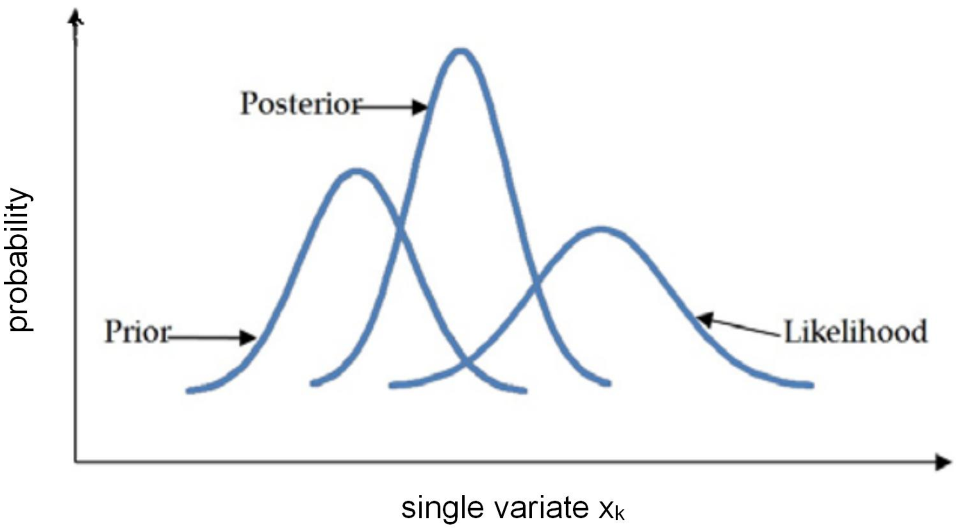

Figure 4.10 Bayesian estimation for importance sampling

Optimal posterior information is shown in Figure 4.10 with Gaussian windowing. Mean correction estimations are deduced based on these probability weightings. However particle filter lacks the prior information as particles are reshaped and weighted with each re-sampling iteration that is needed to control degeneracy. For that matter LSIR method is fused with proposed importance sampling.

4.5.2 Importance Re-Sampling

Benefits of LSIR is visualized in Figure 4.11. Relativity between sampling and re-sampling is covered with LSIR as prior inputs for sampling proposal has known and biased distribution that is established to adjust over high probability regions with a desired distribution. Weights are dismissed in order to adjust a new mean. This method is known to acquire more effective sample size that leads to better results when the model dynamics are designed appropriately. [46]

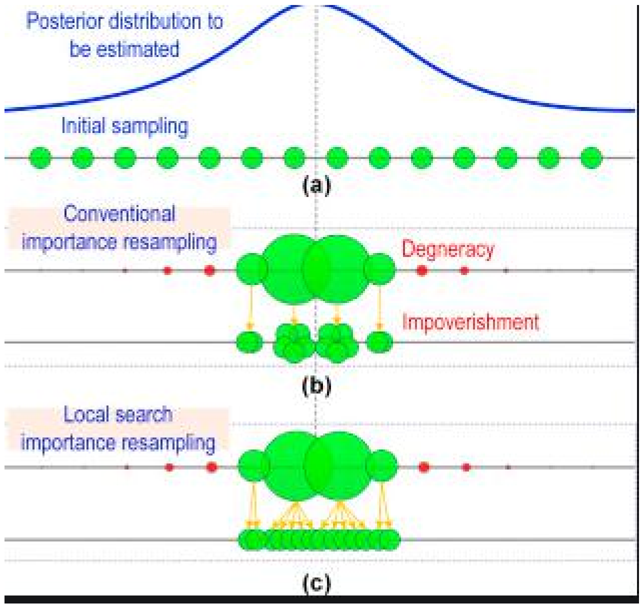

Figure 4.11 Local search importance re-sampling

Another idea behind the administered LSIR method is to keep track of the prior functions based on a statistics threshold that determines particle degeneracy exceeds a limit. The limit is determined by the ratio of the decay between effective sample size and posterior density function variance as both is expected to decay with the same logarithmic ratio as variance breeds from multiplication of two Gaussian. This approach allows the system to re-sample prematurely when unexpected dynamic divergence occurs. This re-sampling trigger will be detailed in Segment 4.5.4 which is defined as error margin factorization.

4.5.3 Adaptive Particle Distribution

Importance weights of particles, $\widetilde{w}_k^n$, that is determined by the proposal function, is presented in Equation (4.23) After passing all the particles through whole proposal calculations, the weights are normalized whose sum is equal to 1 in Equation (4.24) where *N* represents total amount of particles. Weighted means for each state variable, μ_k^{ii} is spotted as the new estimation whose probability density becomes future prediction for the next step if re-sampling does not occur. *ii* denotes state dimension.

$$\widetilde{w}_k^n = \frac{w_{likelihood}^n * w_{k-1}^n}{\sum_s^{k-1} p(y_k \left| x_k^n, x_{k-1}^n \right.) p(x_k \left| x_{k-1}^n \right.)} \tag{4.23}$$

$$w_k^n = \frac{\widetilde{w}_k^n}{\sum_1^N \widetilde{w}_k^n} \tag{4.24}$$

$$\mu_k^{ii} = \sum_n x_k^n * w_k^n \tag{4.25}$$

If re-sampling occurs a new adaptive process noise, that determines the respective distribution after local search for weighted samples, needs to be determined in order to prevent the increase in data association ambiguity caused by ascending distribution. Process noise co-variance can be tuned up so as to compensate for the uncertainties in motion model dynamics for 3-D manoeuvre and acceleration in CTRV case. However validation gates that is formed by particle distribution getting bigger. This phenomenon is controlled by real time adaptive noise co-variances by UKF. In case of particle filter it causes degeneration and impoverishment instead of divergence as the process noise is the weight spread ratio.

Maggio and Cavallaro [47] suggests an adaptive particle distribution with a transition model free system. As system does not propagate particles, the study focuses on portraying uncertainty and covering it with particle distribution. Though, this system is ineffective, it gives an inspiration for adjusting particle distributions based on mapping the difference between measurements and estimations at same time step while a couple previous steps are taken into account. *Qi* represents adaptive process noise in observation coordinates as *i* is the dimension size of observation transition function. Future estimations, μ_k are transformed to spherical coordinates as well.

$$\sqrt{Q_i} = (\sum_{k-2}^{k} \left| y_k^i - \mu_k^i \right|)/3 \tag{4.26}$$

4.5.4 Error Margin Factorization

As we have perfect Gaussian approximation of estimations, the statistics for probability density frame becomes tractable with distribution statistics [48]. Effective sample size, N_{eff} is contemplated in Equation (4.27), described as the distinction between weight impact of particles.

$$N_{eff} = \frac{1}{(\sum_n {w_k^n}^2)} \tag{4.27}$$

In this study, measurement noise co-variance, *R* is designed to be stable as no clutter involved in the design of the system. Adaptive *R* could be established based on kalman like residual calculations in case of a non-stable *R.*

Equation (4.28) computes the mixed Gaussian noise variance σ_k^2 at time step *k*. the value *s* denotes the time steps in which last re-sampling occurs.

$$\sigma_k^2 = \frac{1}{\frac{1}{Q_i + \sigma_{k-1}^2} + \frac{k-s}{R}} \tag{4.28}$$

Confidence interval $z_{\alpha/2}$ defines the effective interval of a Gaussian distribution. It is chosen to be 95% of sample space which corresponds to 2 S.t.d. margin. Equation (4.29) and (4.30) determines the error margin and its threshold respectively on each iteration of algorithm.

$$Err^2 = \frac{(z_{\alpha/2})^2 \sigma_k^2}{N_{eff}} \tag{4.29}$$

$$Err_T{}^2 = \frac{(z_{\alpha/2})^2 R}{N/2} \tag{4.30}$$

When $Err > Err_T$ re-sampling occurs.

As the denominator of the proposal computes data likelihood based on each particle's hypothesis in a Gaussian window, quantization changes the histograms and refine them based on specified number of prior steps *k-s* instead of being a mere coefficient factor. That being said, coefficiency attribute of quantization regulates the weighting spread and allows for higher runs without re-sampling when dynamics are estimated with success. The cases that re-sampling occurs is as below:

Re-sampling occurs when effective sample size are low with respect to adaptive particle distribution, based on data likelihood determined by change in measurement co-variance. This allows the algorithm to keep track on mean even though the measurement are poor until the expected motion loses its validation and can't be met by effective sample size under assigned variance.

If the algorithm converges and stabilize at a weighted mean estimation too fast, system re-samples as error margin exceeds the threshold due to distinction between weighted samples grows sharply. Threshold is exceeded based on high search interval for particles defined as mixed Gaussian variance. This prevents non-precise, diverged track under high measurement noise. In other words, as error prediction is dependant on both weighting and distribution sampling volatility, system invokes re-sampling when the estimations converge to expectation too fast which may not be accurate or the predicted posterior estimation mean diverges too much. These specifications increases the accuracy of desired deterministic re-sampling intervals which also provides accurate quantizations to sampling and re-weighting of particles.

Re-sampling occurs based on possible extension or contraction of measurement error or a possible severe movement that changes the estimated mean. The system re-samples twice when non-linear motion happens and mean estimation reliability is out of date. First re-sampling process resets the quantization element with particles located around prior highly weighted ones. Second one, adjust the particles based on simply maximum likelihood since all prior information on particles and density statistics is lost. This allows for a new particle set to determine a better mean estimation around recent motion dynamics with up-coming measurements. This allows two step fast convergence back to true tracking after abrupt changes in motion happens.

Figure 4.12 implies the significance of quantization determining importance sampling proposal as a better adjustment is made based on prior and maximum likelihood estimations' balance reshaped by quantization factor. That is considered a cheap MAP method without any complex calculations but using auto-regression for reliable measurements.

On the other hand, Figure 4.13 indicates the quantization factor lowering the error margin when poor measurement abruptly occurs. Lower error margin means keeping information alive that supports better data flow to quantization unit. In both noise cases expectation maximization is produced via analyzing priori information trajectory and sequence.

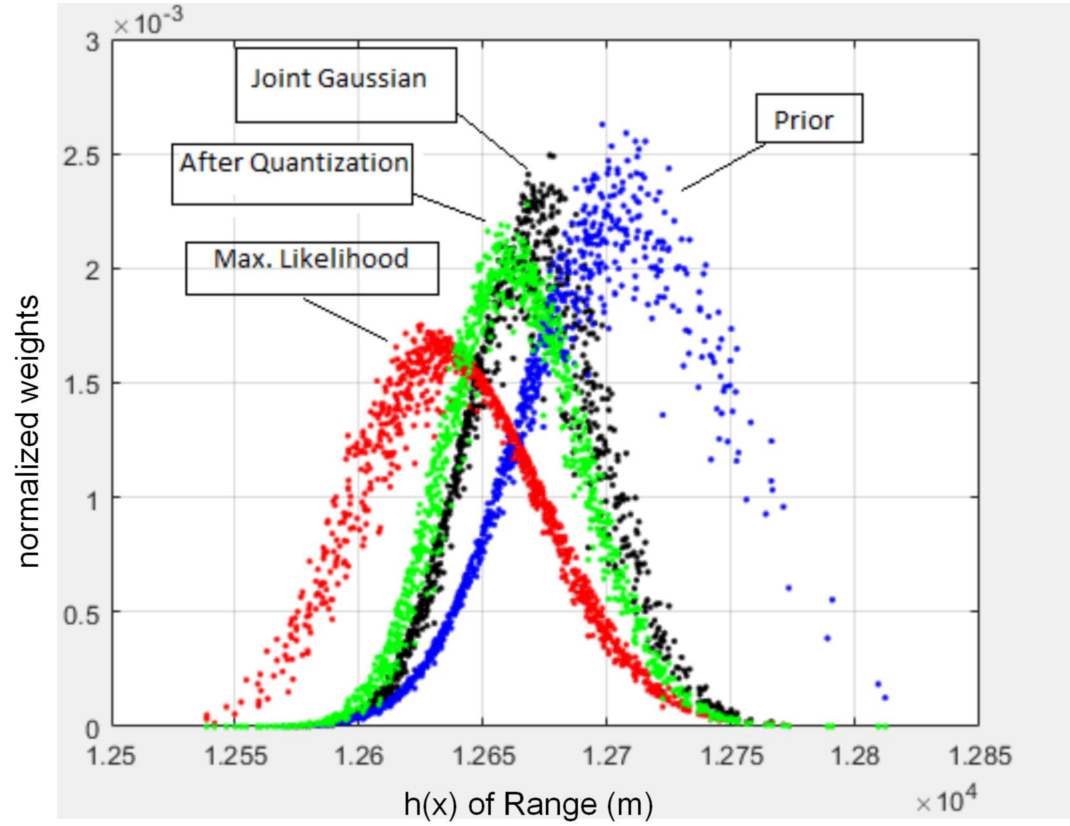

Figure 4.12 Optimal importance sampling proposal based on data likelihood on reliable measurements

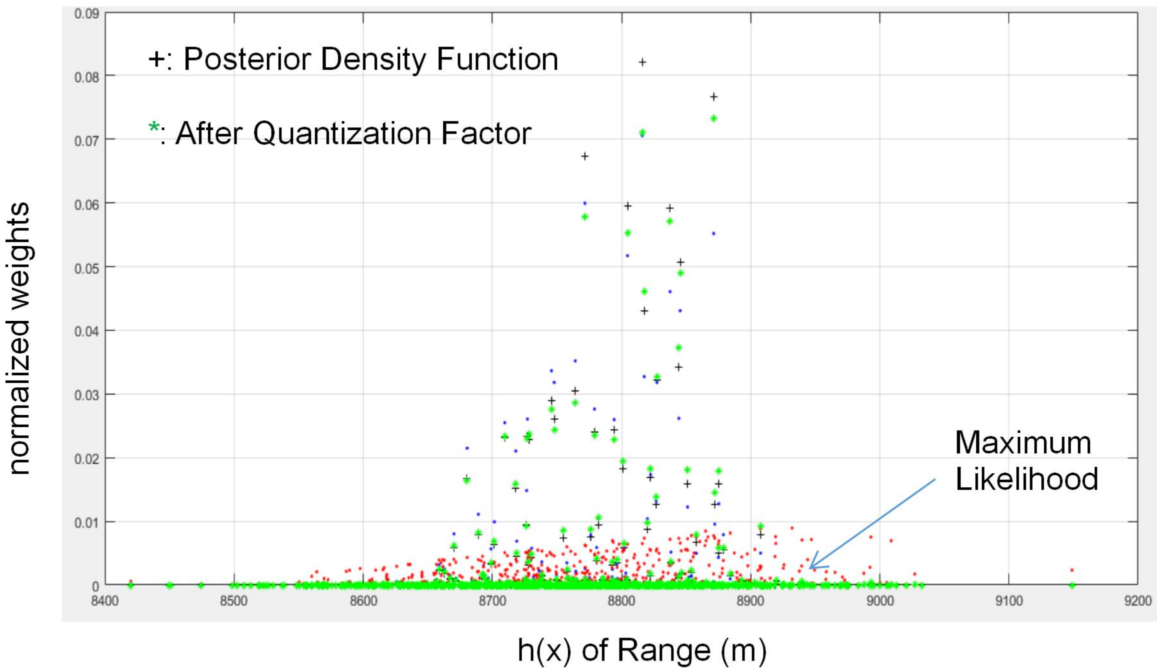

Figure 4.13 Quantization effect on keeping information alive

4.5.5 Pseudo Algorithm

Mentioned complex or default calculations are omitted and described as inscriptions in brackets in pseudo code Figure 4.14.

TAPF for 3-D tracking;

$x_0, P_0 \rightarrow$ **Initialization**

For $n = 1,2,...,N$

Form Samples: $x_0^n = x_0 + \sqrt{P_0} * randn$

$\omega_0^n = 1/N$

EndFor

s=0

Form Transition Functions: (CTRV models *f* and *h* with dimensions *ii* and *i*)

Initial transformed process noise: $Q_i = h(P_0)$

For $k = 1,2,...,T$

For $n = 1,2,...,N$

Propagate Samples: $x_k^n = f(x_{k-1}^n)$

Transform Samples: $\bar{y}_k^n = h(x_k^n)$

Weight ***p(y$_k$|x^n_k,x$^n_{k-1}$)***: $\widetilde{w}_{likelihood}^n = \frac{1}{(2\pi)^{i/2}} \left|R\right|^{-1/2} e^{(-1/2*(y_k - \bar{y}_k^n) R^{-1} (y_k - \bar{y}_k^n)^T)}$

EndFor

Normalize: $w_{likelihood}^n = \frac{\widetilde{w}_{likelihood}^n}{\sum_1^N \widetilde{w}_{likelihood}^n}$

Weight Proposal ***p(x$_k$|y^n_k,x$^n_{k-1}$)***: $\widetilde{w}_k^n = \frac{w_{likelihood}^n * w_{k-1}^n}{\sum_s^{k-1} p(y_k \left| x_k^n, x_{k-1}^n \right.) p(x_k \left| x_{k-1}^n \right.)}$

Normalize: $w_k^n = \frac{\widetilde{w}_k^n}{\sum_1^N \widetilde{w}_k^n}$

Weighted Mean Estimation: $\mu_k^{ii} = \sum_n x_k^n * w_k^n$

Determine Effective Sample Size: $N_{eff} = \dfrac{1}{(\sum_n {w_k^n}^2)}$

(If measurement noise is varying → **Adapt *R* based on residual function**)

Gather Posterior Quantized Density Variance: $\sigma_k^2 = \dfrac{1}{\dfrac{1}{Q_i + \sigma_{k-1}^2} + \dfrac{k-s}{R}}$

(Confidence interval $z_{\alpha/2}$ corresponds to 2σ margin)

Determine Error Margin of Posterior Density: $Err^2 = \dfrac{(z_{\alpha/2})^2 \sigma_k^2}{N_{eff}}$

Determine Error Margin Threshold: ${Err_T}^2 = \dfrac{(z_{\alpha/2})^2 R}{N/2}$

If Err>Err_T

Determine Transformed Process Noise: $\sqrt{Q_i} = (\sum_{k-2}^{k} \left| y_k^i - \mu_k^i \right|)/3$

s=k

For $n = 1,2,...,N$

Resample: $x_k^n \rightarrow$ *(LSIR method)*

Perturbate Samples: $x_k^n = x_k^n + \sqrt{Q} * randn$

$w_k^n = 1/N$

EndFor

EndIf

EndFor

Figure 4.14 Layout of TAPF

A completely stochastic new adaptive algorithm is designed for optimizing 3-D tracking scenarios with less computational cost compared to its rivals, intelligence algorithms. The interconnection between computations of sampling, re-sampling and distribution proposals is provided with error margin factorization which brings the system to a stochastic level that may work on any tracking scenario with given primary model, without entering any deterministic parameter. With the implementation and adaptation of less considered noise distribution and effectiveness of error margin derived from density function, expected improvements are as follows;

Solved impoverishment problems as re-sampling timer is based on predicted error of sampling proposal which is determined by existent distribution spread of particles,

Faster convergence and improved mean tracking based on residual information usage ratio controlled by error margin factor,

High performance under noisy observations as divergence or overfitting problems are appeased with optimal analytic expression of tracking proposal.

5. Results and Discussion

This section consists of comparison of given algorithms under various noise circumstances. Standard deviation of observation noise is denoted as S.t.d. in meters for the following results.

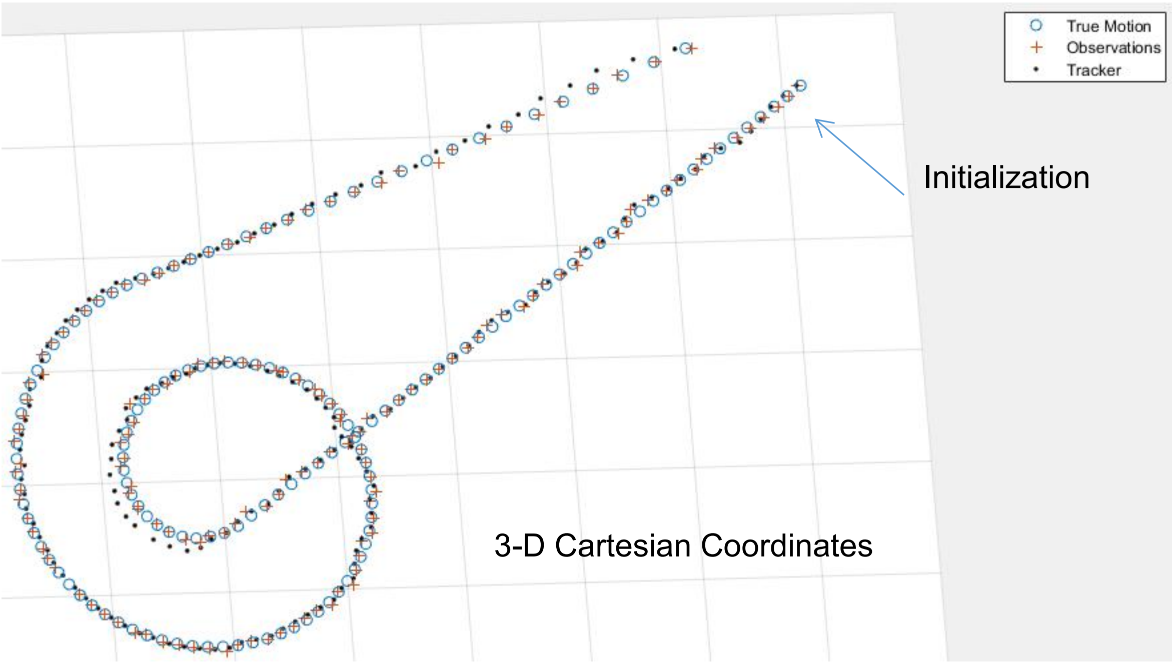

Figure 5.1 UKF results for Target_1 with S.t.d.= 50

Figure 5.1 and Figure 5.2 proves that UKF and PF are practical non-linear track estimators with high quality error approximation and convergence on reliable measurements. PF has better results due to weighting the complete density histogram while UKF uses sigma point cross-correlation. Figure 5.3 represents that the performance of PF is enhanced with the implementation of TAPF algorithm as TAPF has improved techniques for mean estimation. It is shown that TAPF is able to capture non-linear motion sooner and manages a fairly perfect 3-D tracking on true trajectory when there is measurement reliability.

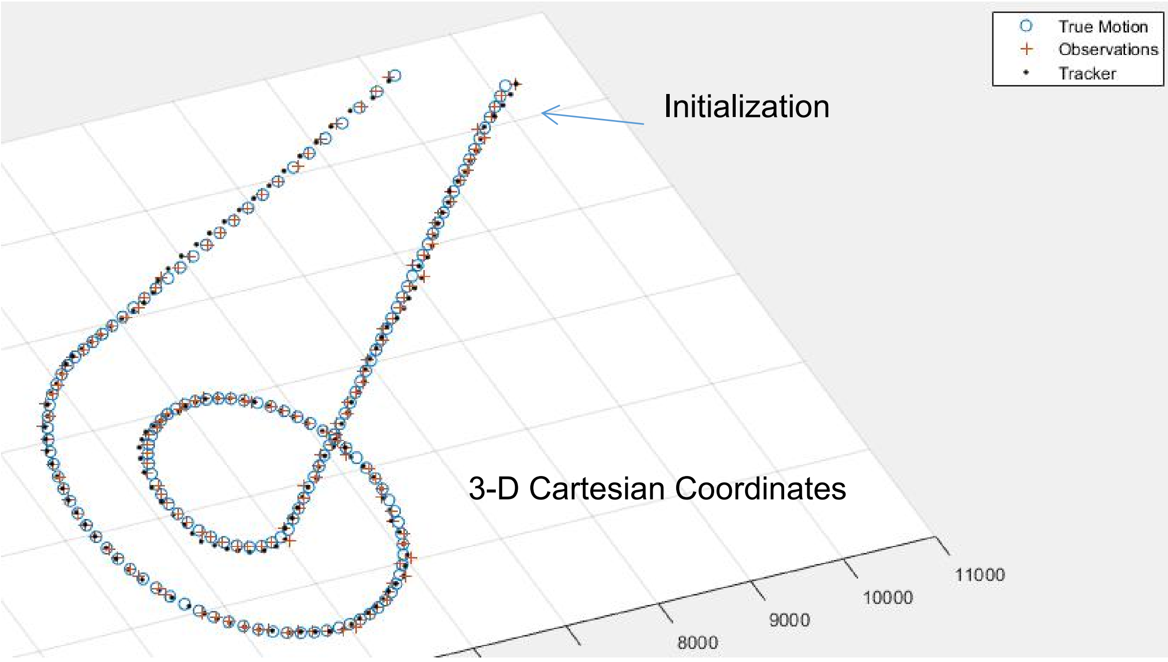

Figure 5.2 PF results for Target_1 with S.t.d.= 50

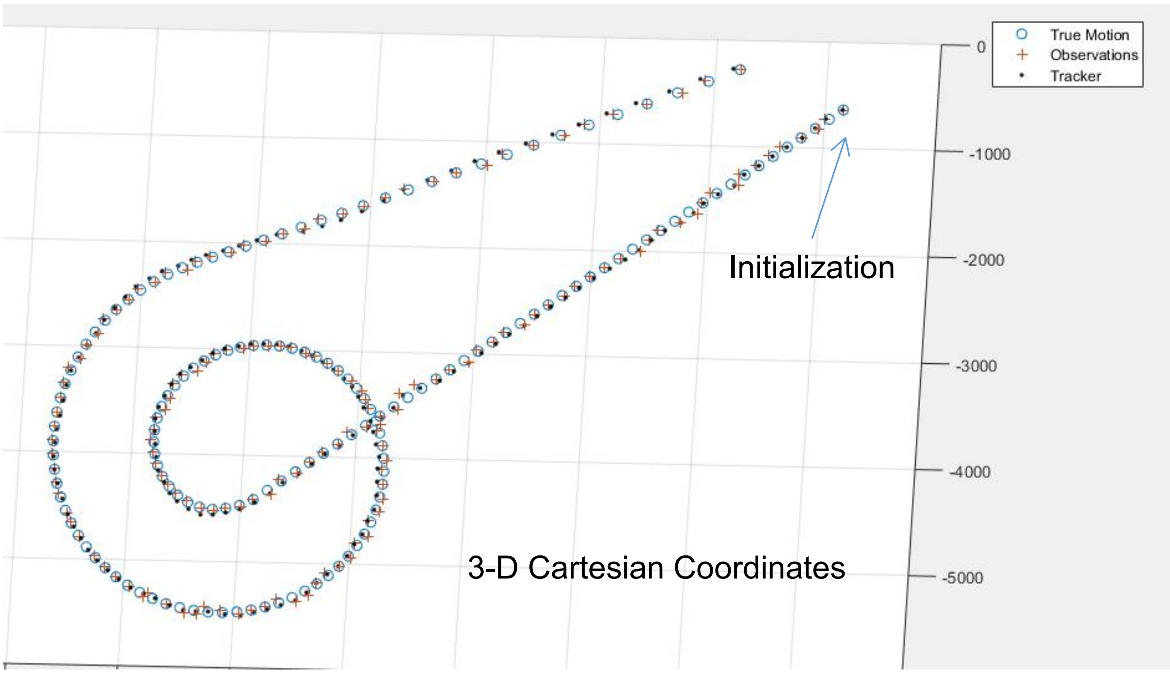

Figure 5.3 TAPF results for Target_1 with S.t.d.= 50

PF slightly loses control on severe changes such as second 43 in target motion dynamics and model mismatch such as acceleration at second 140 as shown in Figure 5.4 for Target scenario 1. TAPF has better manoeuvre control than PF on severe changes when the model satisfies the dynamics of the motion. Figure 5.5, that shows RMSE in meters, proves a slight improvement for TAPF based on effectiveness of particles and efficiency of the system design that adapts to the balance change between estimations and measurements quickly.

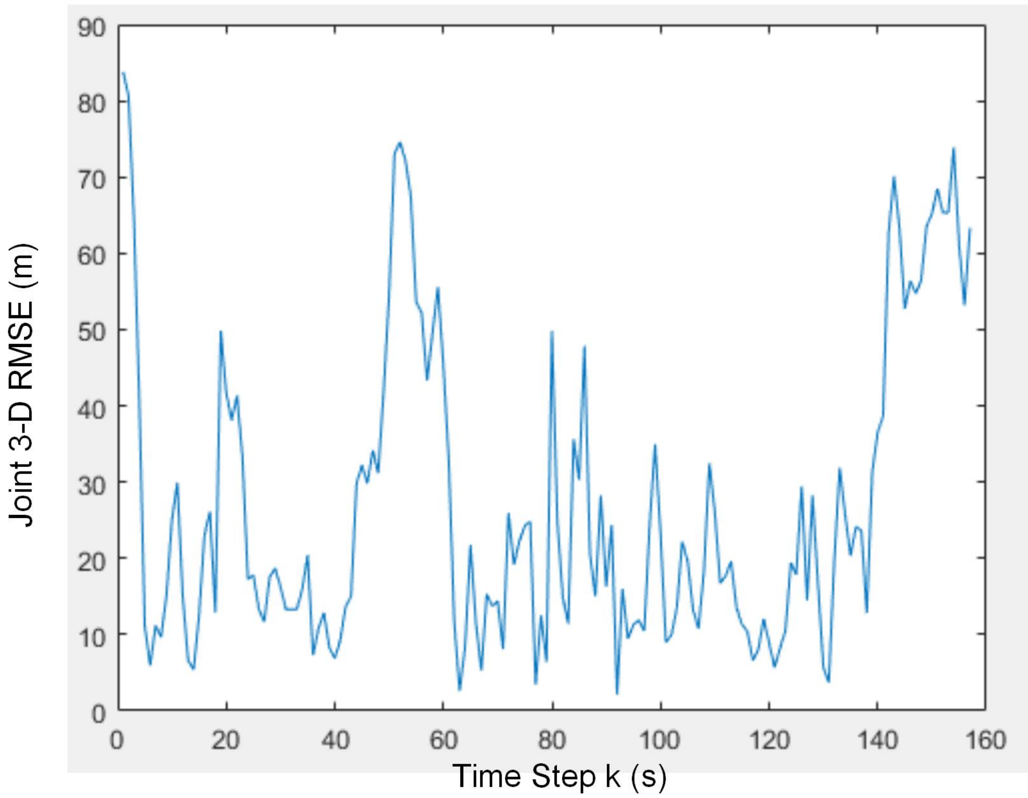

Figure 5.4 PF RMSE values for Target_1 with S.t.d.= 50

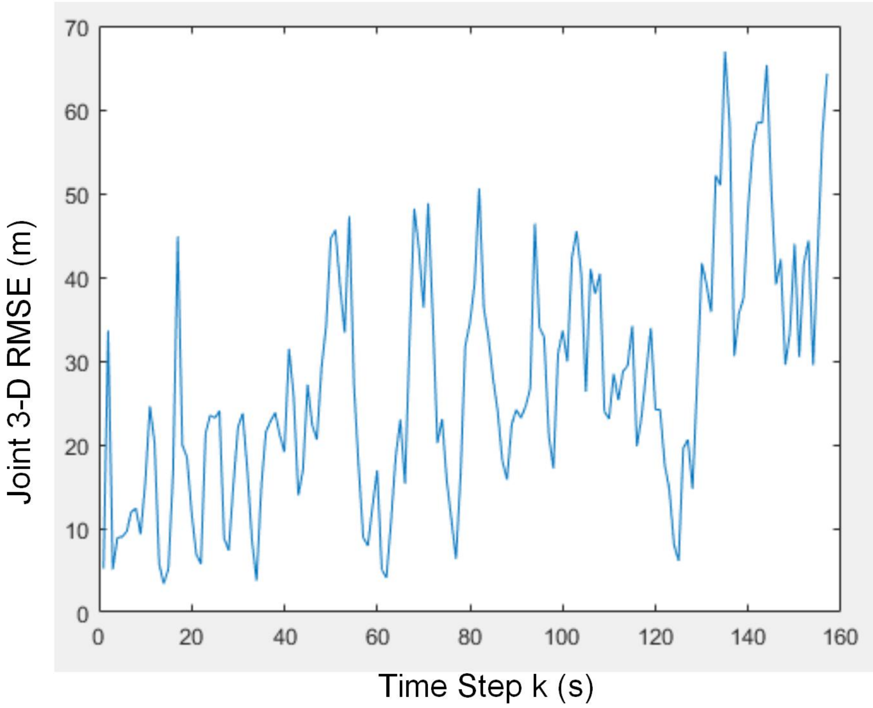

Figure 5.5 TAPF RMSE values for Target_1 with S.t.d.= 50

Figure 5.6 presents that UKF can compensate model mismatch by describing its uncertainty as process noise even for higher order moments that is described as a yaw and pitch angle acceleration uncertainties. PF with 1000 samples, which is lower limit for such 3-D radar tracking, fails to keep accurate track as shown in Figure 5.7. Even though the track system does no diverge, its performance degrades due to degeneration of particles constantly, followed by impoverishment and growing uncertainty gap and distribution between particles.

Figure 5.8 represents that TAPF is an improvement on PF as it resembles kalman based algorithms. However, nature of the filter involves particles that tends to lose effectiveness exponentially under uncertain poor model. UKF works better under model mismatch as a standard. However, model mismatch does not have significance for an emphasize since nowadays model requirements can easily be met with various theoretical or practical implementations for a tracker.

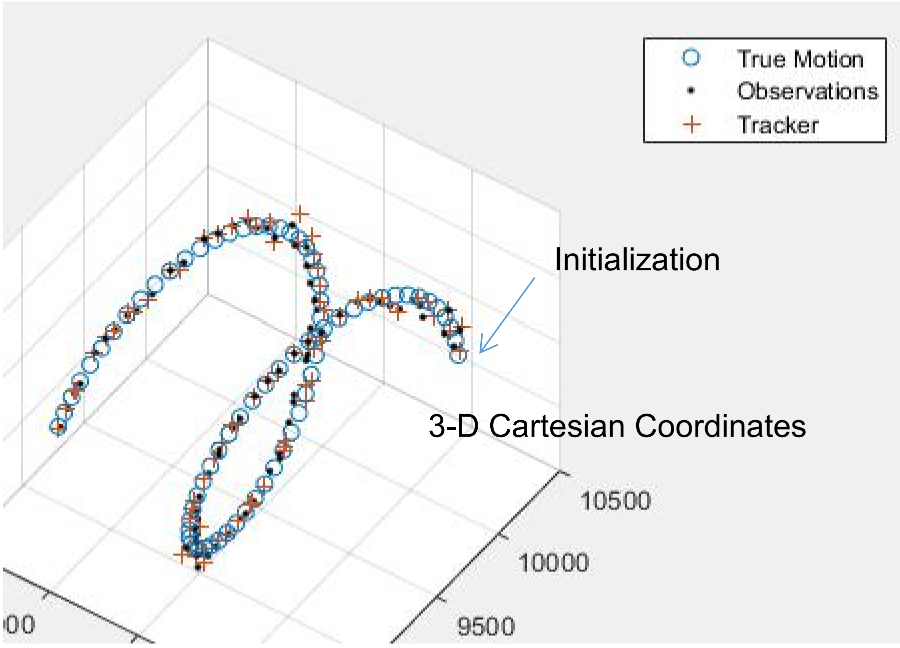

Figure 5.6 UKF results for Target_2 with S.t.d.= 50

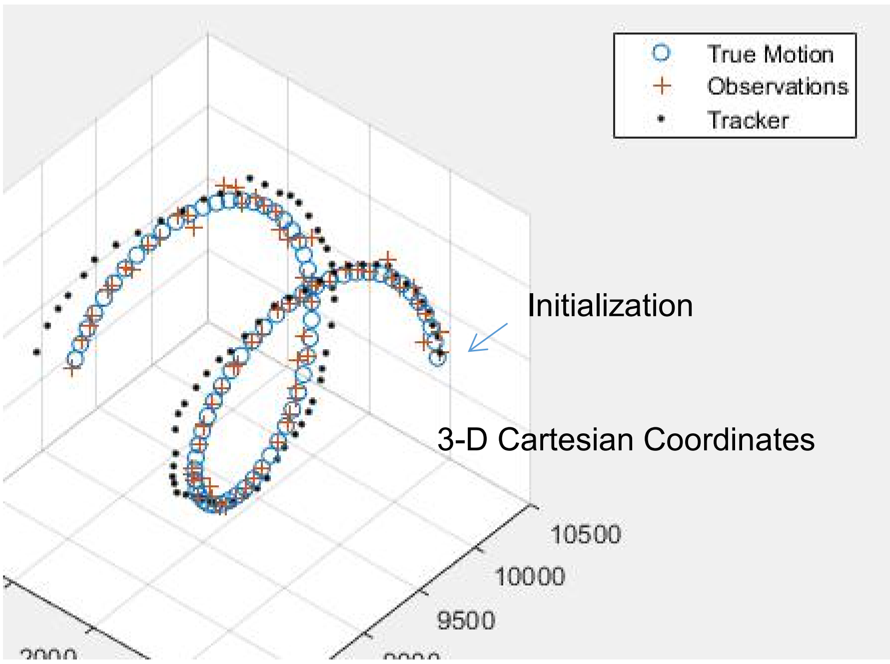

Figure 5.7 PF results for Target_2 with S.t.d.= 50

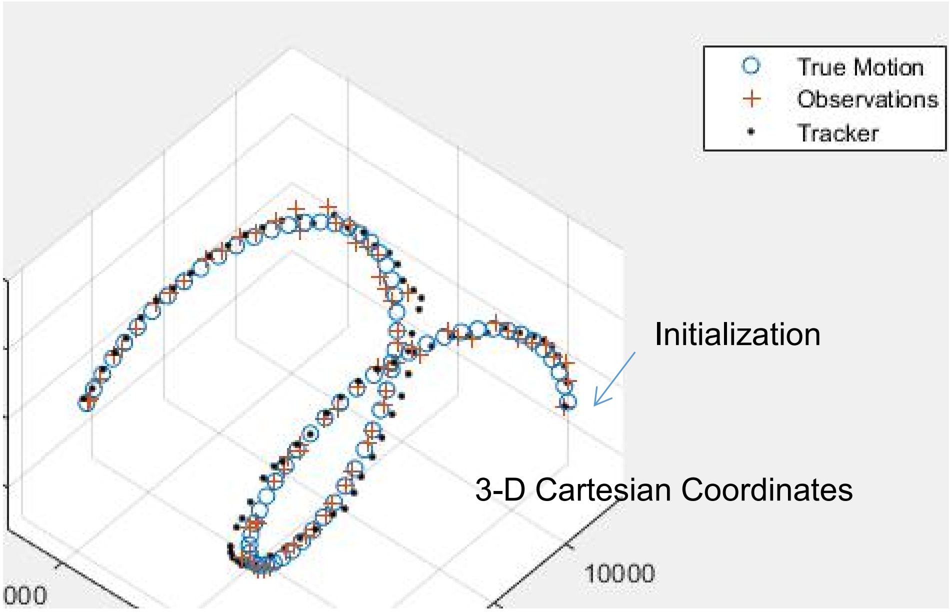

Figure 5.8 TAPF results for Target_2 with S.t.d.= 50

When the measurements are worsened UKF's error co-variance gets bigger which results in convergence to poor measurements in order to be able to keep the track without divergence. Figure 5.9 represents the inefficiency of mean estimations of UKF under noisy measurements. PF has better mean estimations as it considers and weights multiple particles as predictions which is shown in Figure 5.10.

Figure 5.11 yields the leading contribution of this study as it visualize the strength of TAPF which is the perfect mean estimations upon true trajectory under highly noisy measurements. Even though measurements have substantially little meanings, the auto-regressive MAP model and supportive adaptation for that principle makes TAPF robust. The method has the ability to both manage reliable means and converge quickly and adapt to changes and disturbances.

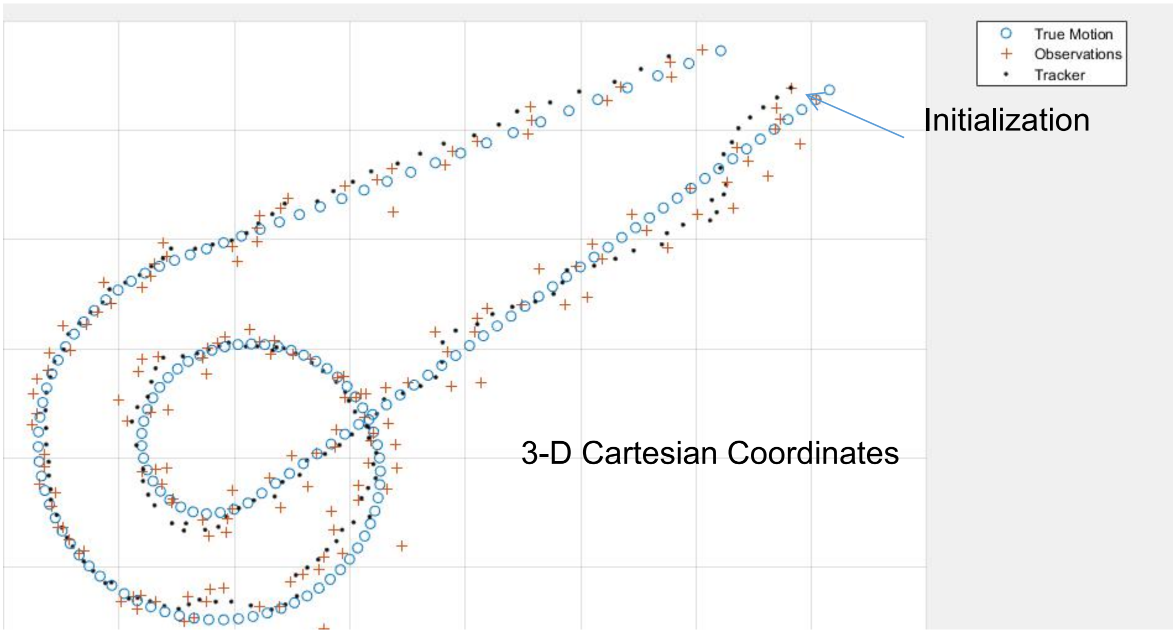

Figure 5.9 UKF results for Target_1 with S.t.d.= 300

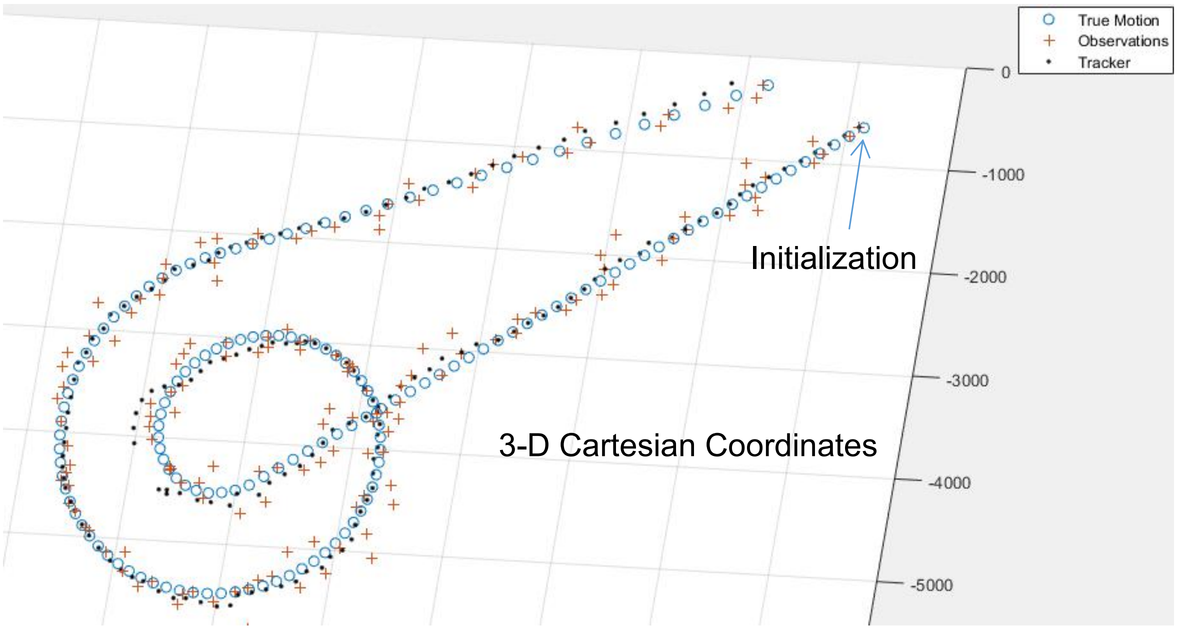

Figure 5.10 PF results for Target_1 with S.t.d.= 300

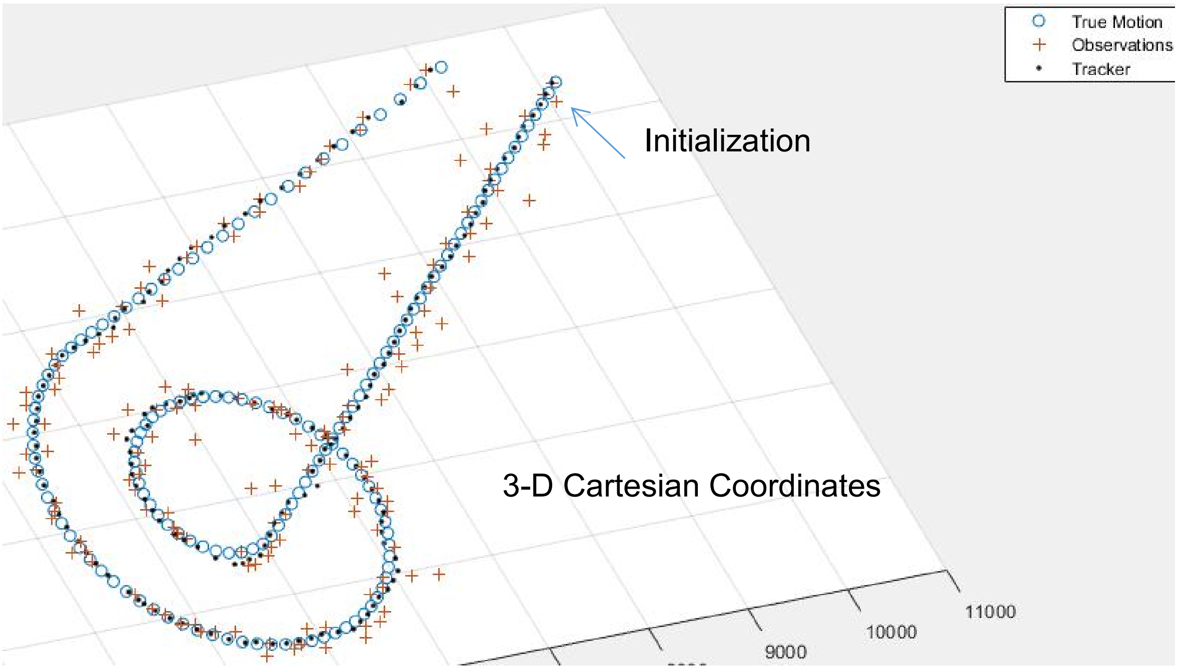

Figure 5.11 TAPF results for Target_1 with S.t.d.= 300

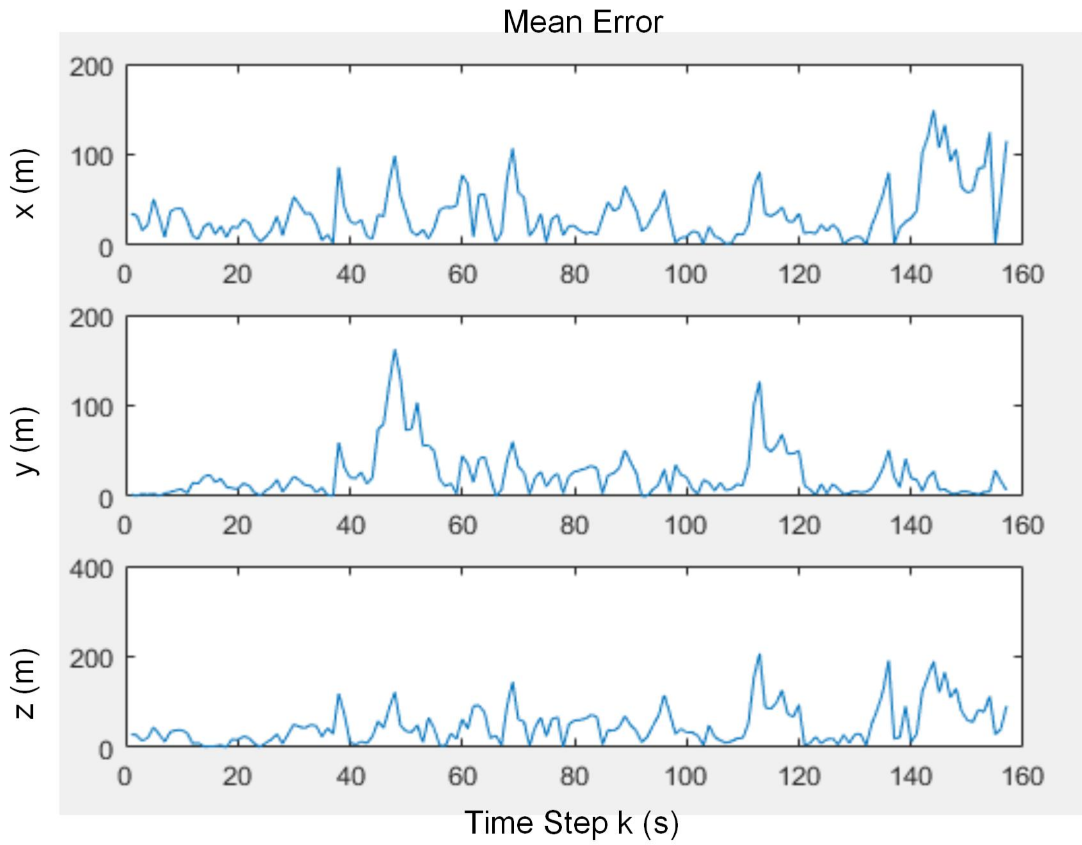

Figure 5.12 TAPF dimensional error for Target_1 with S.t.d.= 300

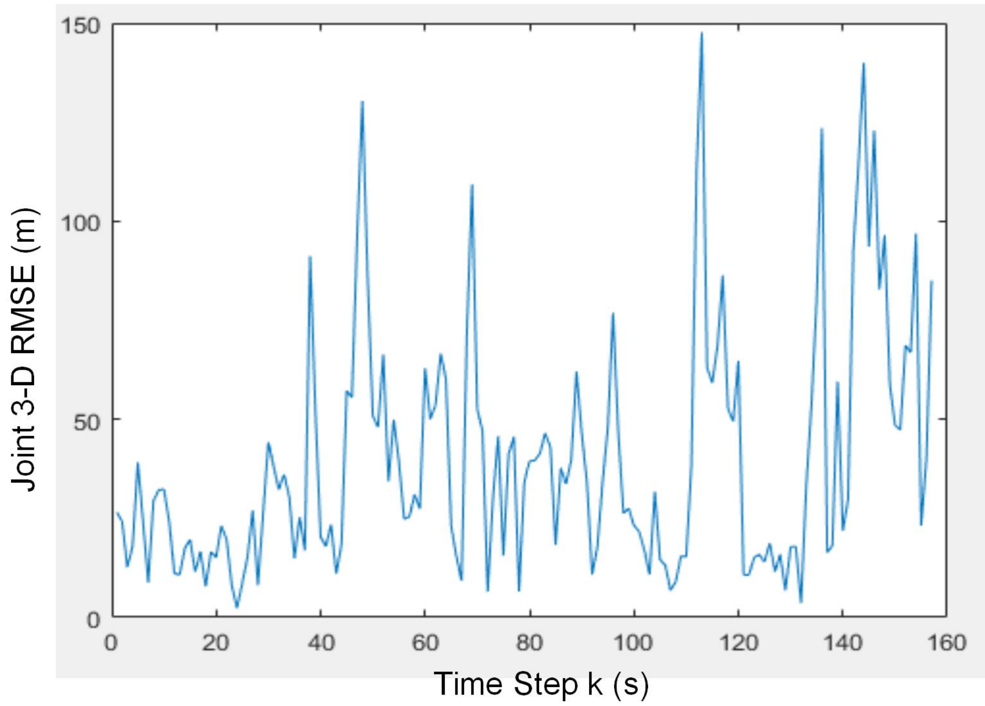

Figure 5.13 TAPF RMSE values for Target_1 with S.t.d.= 300

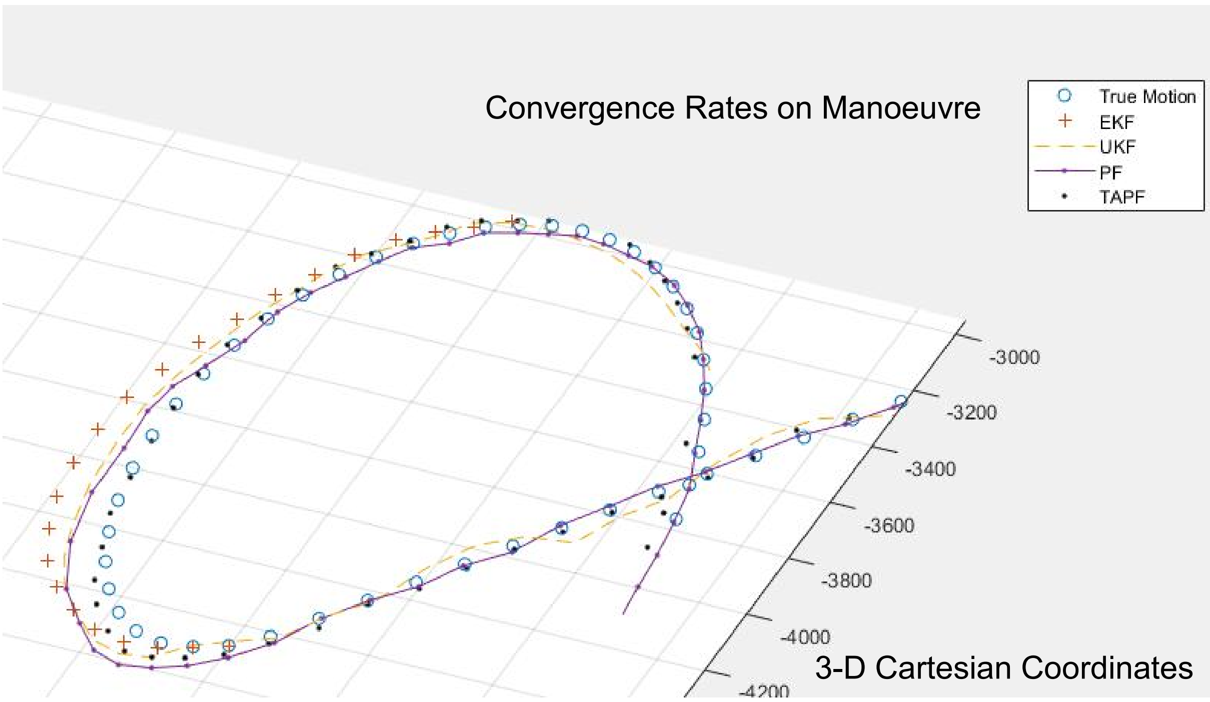

Figure 5.14 Severe manoeuvre filter comparison with S.t.d.=50

The results gives insight to the weaknesses and strengths of different algorithms under same system specifications for comparison and input measurements. Besides visual analogy, Root Mean Square Error (RMSE) estimator is used in order to determine the spread of the expectations which represents the performance with positional error scale in meters. $True_k$ represents the true trajectory of the target motion while *ii*=3 is meant for only zero order moments, Cartesian positions.

$$RMSE_k = \sqrt{\frac{\sum_{ii}(True_k - \mu_k)^2}{ii}} \qquad (4.1)$$

Mean of gathered multiple RMSE for the studied algorithms are given below:

Table 5.1 Performance evaluation of filters based on RMSE values

RMSE Table	Reliable Measures	Poor Measures	Mismatched Model
EKF	88.4	N/A	N/A
UKF	40.8	95.0	39.6
PF	23.2	72.1	123.8
TAPF	18.0	43.3	65.8

Reliable measurements correspond to Target 1 with S.t.d=50 in Table 5.1. Poor measurements represent Target 1 with S.t.d.=300 while Target 2 with S.t.d.=50 is defined as Mismatched Model.

First order linearization has high error when the scale goes higher. Standard EKF should be optimized and fused with other methods in order to achieve a purpose on 3-D target tracking.

On the other hand, UKF imparts better results than PF under the assumption of model mismatch. As sigma point approximation converges to optimal Kalman co-variance predictions, UKF manages to track Target 2 due to superior uncertainty deduction rather than a PF which lacks particle concentration under non-modeled uncertainty and high dimensions. Despite having better results UKF process noise still should be adapted since low significance of priori causes overfitting.

Standard PF has increased performance and better RMSE values when the model is valid but the performance relies on chosen particle distributions since there is limited information on prior conditions and likelihood function output happens to be deterministic when it comes to convergence rate.

As seen from the RMSE results and relevant figures, proposed TAPF method is quite an improvement over the performance of UKF and PF. There is no input to the algorithm except for initial estimation and error which makes the prediction principle of the algorithm, highly stochastic that could be used on every situation. Minimal model mismatches are benign even with the increasing dimensionality since process noise is refreshed when needed.

Another prospect is that the filter has complete knowledge on prior estimations and likelihood of measurement data as the system gives near perfect probability statistics. This opportunity is both used in order to handle process noise, which is less studied in PF applications contrary to UKF, and used to determine optimal solutions. Figure 5.12 and 5.13 shows the over-powered mean track quality of the TAPF compared to a standard particle filter under highly noisy environments up to 900 meters of measurement error. The decrease in TAPF RMSE for poor measurements in Table 5.1 compared to other filters is overwhelming. This enhancement, that is based on perfect mean convergence and adaptation to changes, stands as the main contribution of the proposed TAPF.

Divergence and overfitting trade-off is surmounted with proper re-sampling specifications and weighting if the state and measurement models are well described. Since particle distribution adaption comes from the algorithm itself certain impoverishment problems are minimized with the interconnection and support between adapted methods.

On mild manoeuvre, all algorithms catch up based on CTRV model. Figure 5.14 represents a harsh manoeuvre of Target 1. UKF and PF has similar amount of divergence performances. The curse of reliability is the reason why PF can not orientate suddenly to rapid changes since the particles moving pattern considered these changes unlikely at first glance due to prior information. So, even though PF uses multiple hypothesis, its highly non-linear motion results are similar to sigma point filtering. TAPF adjusts to changes based on rationalized re-sampling timings and corresponding arranged particle distributions with shown superiority (Figure 5.14).

6. Conclusion

Non-linear radar tracking problems for 3-D radar applications was evaluated. Different realistic scenarios with varying noisy measurements and model dynamics were established with RF front end and DSP design based on RDC outputs. Kalman based filters were studied in order to achieve desired solutions for non-linear tracking for non-linear noisy observations.

EKF, UKF and PF are simulated and compared according to their weaknesses and strengths on estimation and error approximation. Based on the problems detected for 3-D tracking, a new method is proposed defined as TAPF in order to achieve enhanced performance solutions. UKF and PF suffer from different varying difficulties such as divergence and overfitting trade-off based on process noise, degeneracy and impoverishment based on inefficiency of particle allocations respectively. TAPF considers a fully stochastic approach with adaptation of filtering methods that has no deterministic processes except for initial inputs.

Bayesian approach which is used in kalman filters is perfected for better estimation of posterior density function as a Gaussian with the dependency of joint noise processes. Kalman resemblance with Gaussian framing achieves an auto-regressive optimal importance sampling proposal that maximize the expectations with the ability to analyze output function statistically. The method is fused with LSIR in order to diminish the effects of information loss. A proposed particle adjustment based on adaptive process noise is formed. As the sampling, re-sampling and particle distribution methods implemented, supports each other by maximizing their tractability and minimizing the problems they suffer, an error margin factorization is established in order to relate and keep the mentioned methods intact jointly. This factorization obtains information from posterior quantized density function to improve the reliability of the estimations and initiates re-sampling and changes the particle dynamics based on the information gathered.

Results indicated that TAPF provides slight improvement on track estimation for reliable measurements. TAPF responds faster to highly non-linear motions such as severe manoeuvring and converges better rather than UKF and PF. The main contribution of the study is that TAPF provides nearly perfect mean estimations and converges faster than expected on severe changes considering high measurement noise in the system based on error margin intellect.

As TAPF benefits from prior information instead of any external methods, the computational cost is no more than a standard naive PF. So, TAPF may be an alternative option for costly intelligence algorithms for MAP estimation. Another possible future improvement can be the particle distribution optimization based on expectations. Chen, Tharmarasa, Pelletier, and Kirubarajan [49] suggests clutter estimation to be integrated in track processor. In order to estimate clutter spatial density, a recursive maximum likelihood method is derived considering the clutter model. As TAPF works extraordinarily well on high noise, a non-Gaussian clutter model and histogram could be attempted with the Gaussian mixture proposal.

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